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Artículos científicos

Instrumento para evaluar los conocimientos matemáticos previos para la enseñanza del concepto de *límite* durante la pandemia SARS-CoV-2

Instrument for evaluating prior mathematical knowledge for the teaching of Limit concept during the SARS-CoV-2 pandemic

Instrumento para avaliar o conhecimento matemático prévio para o ensino do conceito de limite durante a pandemia de SARS-CoV-2

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Resumen

El objetivo del presente trabajo fue diseñar un instrumento que permitiera determinar, en el marco de la actual emergencia sanitaria generada por el virus SARS-CoV-2, los conocimientos matemáticos previos en torno al concepto *límite* de alumnos del bachillerato de Ingeniería y Arquitectura (sección 1) del Colegio Primitivo y Nacional de San Nicolás de Hidalgo. Para el diseño del instrumento se siguieron las cuatro fases de construcción que establece Soriano Rodríguez (2015), es decir, 1) objetivos, teoría y constructo, 2) validación juicio de expertos, 3) prueba piloto y 4) validación psicométrica. Los resultados de esa última fase demuestran que el instrumento se puede reproducir en diferentes muestras sin que constituyan un gran sesgo por parte de los participantes. Además, los puntajes de cada pregunta fueron consistentes con el puntaje total del instrumento. En conclusión, se puede afirmar que esta puede ser una herramienta muy útil no solo para detectar los conocimientos previos que los alumnos tienen en cuanto al cálculo diferencial, sino principalmente para crear estrategias didácticas que permitan atender esas debilidades a partir de los estilos de aprendizaje de los estudiantes.

Palabras clave: cálculo diferencial, enseñanza emergente, exploración, diagnóstico, límite.

Abstract

The technology that we have today is not conceivable without one of the pillar concepts in the construction of the Differential Calculus, the Limit. An important concept that, since its "discovery or invention" by Newton and Leibniz, has created difficulties for many mathematicians to understand it, it was not until the arrival of Cauchy that he defined it in a more practical way, but not for students, since various studies refer to the difficulty it has the study of Limit as well as the misconceptions that the student gives it and the little success on the part of the students in their understanding in which didactic strategies have been carried out for their understanding, all in a face-to-face setting. However, for reasons of SARS-CoV-2, the situation is even more difficult since everything is framed in an emerging teaching. Therefore, it is decided to design and create a diagnostic instrument of the student's previous knowledge that precedes the learning of Differential Calculus Limit in such a way that it measures, characterizes and synthetically explores both the procedural and conceptual skills of the cognitive processes of arithmetic and algebra, such that it allows to have enough constructs to understand the concept of limit. The instrument is validated in four phases in which the judgment of experts and the psychometric processes that allow the generation of scientific evidence are contemplated. This instrument conjectures the weaknesses that students may have and provides the teacher with a horizon to decide the didactic methodology to address the topic. With this result, it joins the efforts of various researchers in educational mathematics who seek to improve both the teaching and learning of the concept of limit.

Keywords: differential calculus, emerging learning, previous knowledge, diagnosis, limit.

Resumo

O objetivo deste trabalho foi delinear um instrumento que permitisse determinar, no quadro da atual emergência sanitária gerada pelo vírus SARS-CoV-2, o conhecimento matemático prévio em torno do conceito de limite de alunos do Bacharelado em Engenharia e. Arquitetura (seção 1) da Escola Primitiva e Nacional de San Nicolás de Hidalgo. Para a concepção do instrumento, foram seguidas as quatro fases de construção estabelecidas por Soriano Rodríguez (2015), ou seja, 1) objetivos, teoria e construto, 2) validação do julgamento pericial, 3) teste piloto e 4) validação psicométrica. Os resultados desta última fase mostram que o instrumento pode ser reproduzido em diferentes amostras sem constituir um grande viés por parte dos participantes. Além disso, as pontuações de cada questão foram consistentes com a pontuação total do instrumento. En conclusión, se puede afirmar que esta puede ser una herramienta muy útil no solo para detectar los conocimientos previos que los alumnos tienen en cuanto al cálculo diferencial, sino principalmente para crear estrategias didácticas que permitan atender esas debilidades a partir de los estilos de aprendizaje de os estudantes.

Palavras-chave: cálculo diferencial, ensino emergente, exploração, diagnóstico, limite.

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Introduction

Due to the problems caused by covid-19, we are currently in a stage of reconstruction of teaching and learning, which has had to rely on emergency remote teaching strategies to meet the educational needs of the population. As committed by Hodges, Moore, Lockee, Trust and Bond (2020), the main objective has been to provide access to instruction, as well as pedagogical supports that allow the continuity of educational activities and training for teachers to promote collaborative work and teaching in virtual environments.

However, according to data from the United Nations Educational, Scientific and Cultural Organization (UNESCO, 2020), the current health emergency has caused that almost 85% of students worldwide cannot carry out their academic activities normally. mainly due to problems related to poor access to communication technologies. In Mexico, for example, according to the National Survey on the Availability and Use of Information

Technologies in Homes (Endutih) (National Institute of Geography and Information Statistics [Inegi], 2018), only 47% of the inhabitants have a connection to Internet, 52.4 million people live in poverty and 9.3 million in extreme poverty. In addition, 21.1 million people suffer from educational backwardness (National Evaluation Council [Coneval], 2018).

This reality is worrying because it tends to affect vulnerable populations more harshly (Rojas Maldonado, 2020a), who tend to remain awaiting humanitarian initiatives carried out by the State. In the case of Mexico, the government's educational policy strategies to address the health emergency have focused on ensuring that school activities are maintained virtually. However, everyday life has shown that both for teachers and for students the teaching and learning process has been considerably hampered.

For this reason, the objective of this work was to design an instrument that would allow to determine, within the framework of the current health emergency generated by the SARS-CoV-2 virus, the previous mathematical knowledge around the limit concept of students of the Engineering Baccalaureate and Architecture (section 1) of the Colegio Primitivo y Nacional de San Nicolás de Hidalgo.

The concept of limit is essential for the construction of the differential calculus, to successfully face university studies related to mathematics and to promote the understanding and technological development on which our civilization is currently based (Cottrill et al., 1996), and it has been approached by some authors (Artigue, 1998; Cornu, 1991), who have demonstrated the incorrect appraisals of the students about this concept. To improve their understanding, various strategies have been applied that have been supported by technological tools, which have allowed achieving slightly higher results than those achieved with traditional teaching. (Rojas Maldonado, 2015, 2016, 2019).

Theoretical framework and background

In general, it can be affirmed that although there are investigations that could be linked to the main objective of this work, in reality there are few that focus on the study of students' prior knowledge about the limit concept in the middle of a situation as particular as that generated by the current pandemic. Even so, inquiries such as those of Mota Villegas and Valles Pereira (2015) can be rescued, who not only explain the importance of the structured study of prior mathematical knowledge of students who begin at university, but also demonstrate how incomplete they can be the instruments used today to measure such knowledge.

For his part, Urbina (2009) affirms that the evaluation of initial knowledge helps the teacher to determine the degree of depth with which a topic should be treated, as well as the strategies that can be used so that the student assimilates the new meanings of the object of learning offered. In this regard, the aforementioned author states the following:

> During the last 20 years, research in mathematics education has been marked by the constructivist paradigm. The key ideas of this paradigm come from or have their roots in the research of many authors, among which stand out: Piaget, Wallon, Vygotsky, Bruner, Dewey, Gagné, Ausubel, Novak and Henesian, among others. All of them have agreed that learning any school content supposes from the constructivist conception, attributing a meaning and constructing the meanings implied in said content, and that this construction is not carried out starting from scratch (Urbina, 2009, p. 2).

In this sense, Pino-Fan, Godino and Vincenç (2013) design and apply an instrument to explore the epistemic facet of the didactic-mathematical knowledge of future teachers on the derivative. These authors demonstrate that the mathematical objects, their meanings and the processes identified in the plausible solutions of the tasks are adapted to the mathematical objects, their meanings and the processes involved in the cognitive configurations (cognitive ontosemiotic analysis) associated with the teachers' responses. initial training.

Likewise, Ndjatchi (2019) designs an instrument to determine the previous mathematical knowledge and weaknesses of a student of the computer systems career before taking the complex numbers course. Similarly, Morantes Moncada, Dugarte Peña and Herrera Díaz (2019) —from the review of planning skills, control and review of learning activities— characterize the profile of a strategic student for the analysis of differential calculus in the topic functions and their graphs.

However, numerous investigations (Artigue, 1988; Rojas Maldonado, 2018, 2019, 2020b; Tall, 1995) indicate little capacity to establish a symbiosis between geometric and symbolic representativity on the part of students, an essential skill for the abstraction of mathematics (Ndjatchi, 2019). Peña, Murillo, Rodríguez, Cedillo and Green Arrechavala (2014) explain it this way:

> The errors of the students are not accidental, they are based on previous knowledge and experiences, and are motivated by different didactic, epistemological, cognitive or attitudinal causes. In this way, we observe that errors appear in students' work mainly when they are faced with new knowledge that forces them to review or restructure what they already know. (p. 48).

During teaching and learning, as Ndjatchi (2019) mentions, “difficulties may arise related to their own nature, others due to the teaching circumstances, another of the teachers and their didactic methodology” (p. 309), which is consistent with that indicated by authors such as Artigue (1988), Cottrill et al. (1996), Rojas Maldonado (2018, 2019) and Tall (1995). Therefore, before choosing a didactic methodology to develop the educational task, the difficulties that students face must be known in detail.

> It is appropriate to take into account the initial knowledge of the students and to establish a coherent relationship between what the students know and the new knowledge. Otherwise, unnecessary difficulties and a lack of confidence of the students to face the new knowledge would be generated. (Urbina, 2009).

In this regard, it should be taken into account that meaningful learning arises when the relational anchor between an existing content and a new one is clear and available in the student's cognitive structure (Ausubel, 1983), so it can establish similarities and differences of non-arbitrary form. Therefore, Ausubel (1983) specifies the following:

> The acquisition of new information is highly dependent on pertinent ideas that already exist in the cognitive structure, and meaningful learning in humans occurs through an interaction of new information with relevant ideas that already exist in the cognitive structure. (p. 7).

This prior knowledge, as López Recacha (2009) explains, is personally constructed by each individual while interacting with people or objects, and should be used to avoid rote learning, which is far from Ausubel's meaningful learning. It is true that in many cases prior knowledge can be temporarily forgotten, but with a quick reminder students are able to regain meaning and implications.

Now, regarding the instruments used to determine the students' prior knowledge, first, the conceptualization of the construct to be evaluated must be clear, as well as its execution standards and its respective evaluation indicators (García, 2018). As Soriano Rodríguez (2015) comments, any measurement process in an investigation must transcend from reliability to validity. For this, the instrument must be the product of an articulation between paradigm, epistemology, theoretical perspective, methodology and techniques for data collection and analysis. In this sense, an instrument is valid if it measures what it claims to measure, while reliability focuses on the probability of obtaining the same results.

Finally, one aspect sought by those conducting research is the need for the selected instrument to be optimal for collecting the data and to have a high level of reliability, especially when they base their validity on verbal information of perceptions, feelings, attitudes or behaviors (Tuapanta Dacto, Duque Vaca, A. and Mena Reinoso, 2017), hence Jiménez Alfaro and Montero Rojas (2013) consider that their design must be supported by a process of continuous perfection.

Objective

Design an instrument to determine, within the framework of the current health emergency generated by the SARS-CoV-2 virus, the previous mathematical knowledge around the limit concept of students of the Engineering and Architecture baccalaureate (section 1) of the Primitivo y Nacional School of San Nicolás de Hidalgo.

Methodology

For the design of the instrument, the four construction phases established by Soriano Rodríguez (2015) were followed, that is, 1) objectives, theory and construct, 2) validation of expert judgment, 3) pilot test and 4) psychometric validation, which Are detailed below:

Developing

Phase 1. Objectives, theory and construct

As part of the design process, the analysis of the academic program of the Differential Calculus subject was carried out in order to identify the minimum academic needs of the limit thematic unit. Subsequently, a bank of problems was created that have been studied in various investigations focused on the didactics of calculus and that have been considered in admission exams at the professional level. Then, the items that satisfied the criteria of the epistemic facet were selected: 1) common knowledge 2) specialized knowledge and 3) expanded knowledge (Loewenberg Ball, Hoover Thames and Geoffrey, 2008; Loewenberg Ball, Theule Lubienski and Spangler Mewborn, 2001) .

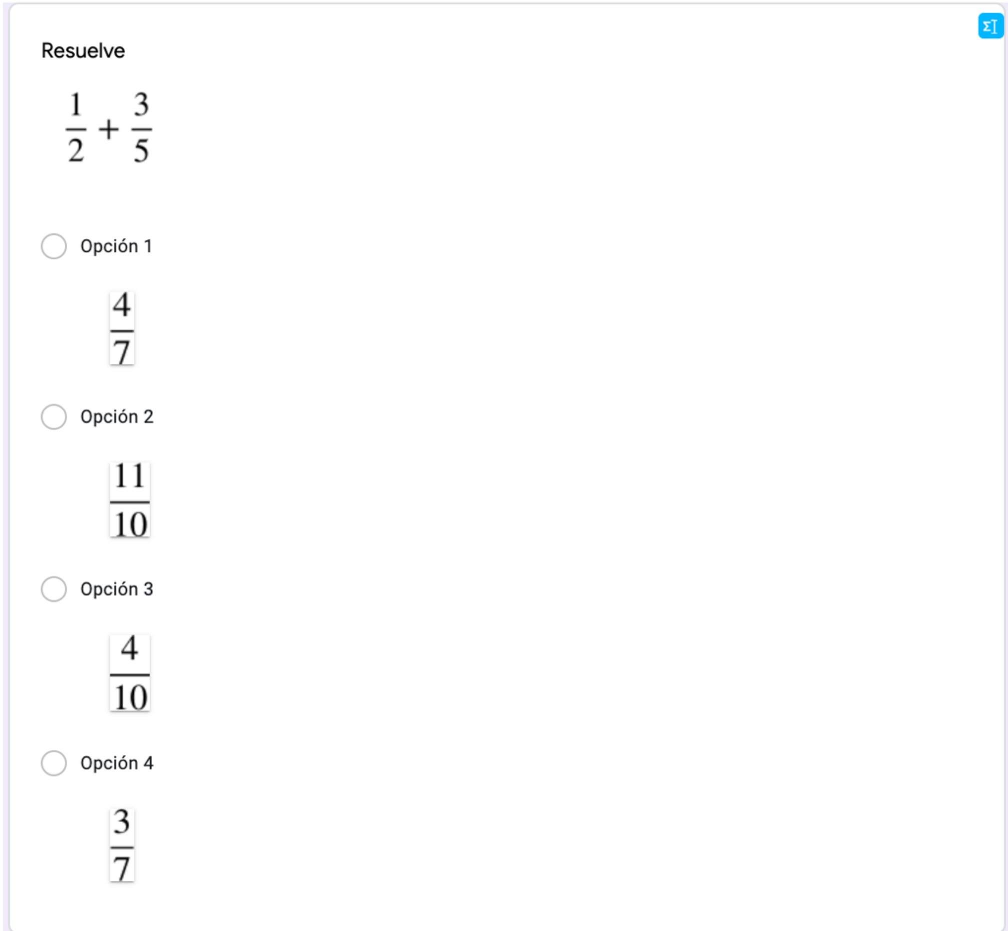
The instrument was structured to reflect previous academic knowledge, such as arithmetic ability and algebraic ability, in order to identify problem solving and the construction of mathematical knowledge.

The design of the instrument was focused on considering different levels of algebrization and arithmeticization (Godino et al., 2015), since awareness of the progression of learning was sought, as well as alerting the teacher of the pertinent difficulties (conceptual, propositional, procedural and argumentative). To do this, we proceeded as follows (Godino *et al*., 2015):

> We classify the content to be evaluated according to two variables: algebraic content and didactic content. For the algebraic content variable, we consider three values or categories, in which various subcategories can be distinguished:
>
> - Structures: Equivalence relation; properties of operations, equations, etc.
>
> - Functions: Arithmetic patterns, geometric patterns; linear, affine, quadratic function, etc.
>
> - Modeling: Context problems solved by proposing equations or functional relationships.

Due to the pandemic, the instrument was applied through Google Forms, a tool that allowed items to be ordered randomly and answered in a limited time. In figure 1 the common mathematical knowledge is structured, focused on the epistemic facet, that is, recognition of algebraic and arithmetic processes (concepts, procedures, properties, generalization, etc.).

Figura 1. Conocimiento matemático común



Resuelve

$$\frac{1}{2}+\frac{3}{5}$$

Opción 1 $\frac{4}{7}$

Opción 2 $\frac{11}{10}$

Opción 3 $\frac{4}{10}$

Opción 4 $\frac{3}{7}$

Fuente: Elaboración propia

At the previous level, the students did not require a capacity for analysis or reflection, but rather follow a procedural mechanism.

On the other hand, regarding the subcategories of classification of content, structures and functions of the epistemic facet of specialized knowledge, figure 2 shows that although there is a procedure for solving the problem, it actually requires a capacity for analysis and reasoning, as well as the identification of different variables that involve properties, arguments and procedures.

> This notion favors not only the systematic identification of different resolution procedures, modes of expression, concepts and properties that are put into play in its formulation, but also the identification of arguments or justifications of the procedures and properties. (Pino-Fan, Godino, Castro y Font, 2012).

Figura 2. Conocimiento especializado

En un puesto de verduras del mercado, un cliente realizó la siguiente compra: 22.5 Kg. de cebolla, 13.6 Kg. de Jitomate, 13.3 Kg. de chile serrano, 18,8 Kg. de papas, 17.1 Kg. de Zanahoria y 5 piezas de repollo.El dueño del puesto quiere saber cuanto ganó en esa venta, si: El Kg. de jitomate le costó $ 8.50 y lo vendió a $ 14.00, el Kg. de cebolla lo compró a $4.60 y lo vendió a $ 7.60, el Kg. de papas lo compró a $ 2.60 y lo vendió a $ 4.30, el Kg. de zanahoria lo compró a $ 3.80 y lo vendió a $ 6.30, el Kg. de chile lo compró en $2.50 y lo vendió en $ 4.10 y la pieza de repollo le costó a $ 4.20 y la vendió en $ 7.00 ¿Cuál es su ganancia?

- 43.30
- 639.50
- 252.29
- 387.21

Fuente: Elaboración propia

Performing an epistemic analysis of figure 2, the following can be noted:

Representation process

Elements can be identified through differentiation, which the student must be able to distinguish. Likewise, she must know the process to which she must resort to establish a one-to-one connection that allows her to locate and select the variables in a mathematical expression; for example, the variable X will represent tomatoes, Z will represent carrots, etc., which concerns a symbolic or notational representativeness, indicating the correlation between the two items in their respective moments. This provides information that could be used by both the teacher to adjust his teaching method and the student to identify a mathematical or logical weakness.

The representation process is a fundamental part of problem solving, since its disarticulation leads to synthesis, which is represented through a graph or symbology (restructuring of the phenomenon) which, in the case that concerns us, is the limit.

Composition-decomposition process

Once the variables of the previous process have been identified, the elements that respond to similarities can be discerned.

Figura 3. Proceso de composición-descomposición

Si x equivale a jitomate

$$13.6x = (\$14 - \$8.5)$$

Fuente: Elaboración propia

As indicated by Pino-Fan et al. (2012), "similar analyzes can be carried out with other resolutions of the task, which bring into play other procedures and justifications that involve different linguistic elements, concepts and propositions. In this sense, the meaning processes do not have unique configurations "(p. 431). Later, in figure 4, this process is also clearly shown, which requires a greater capacity for synthesis, analysis and reflection.

Process of particularization-generalization

Figure 3 started from a particular fact that can well be generalized for each of the elements mentioned in figure 2, that is, it is generalized for onions, chili peppers, etc. It is to be noted that it is closely linked to the transition from common and advanced knowledge.

Materialization-idealization process

In this process, specific knowledge about some area of mathematics that requires greater capacity for analysis and reflection on the part of the student is explored, as shown in figure 4, where ideas or knowledge that cause dissonance can often be opposed.

Figura 4. Proceso de materialización-idealización

El iPhone 11 Pro Max tiene un costo de $27,499 pesos con IVA incluido. Si el IVA es del 16%. ¿A cuánto asciende el monto correspondiente de IVA?

- $4399.84
- $3,792.96
- $31,898.84
- $4231.73

Fuente: Elaboración propia

As suggested by Godino et al. (2015), in each item we include the expected solution, as well as three incorrect distracting responses: the objects and processes that are manifested for the resolution, the level of difficulty corresponding to the academic activity and the categories of didactic-mathematical knowledge involved according to Table 1.

Tabla 1. Categorías de conocimiento didáctico-matemático

Contenido didáctico	Contenido algebraico					
	Estructuras (E)		*Funciones (F)*		*Modelización (M)*	
	Primaria	*Avanzado*	*Primaria*	*Avanzado*	*Primaria*	*Avanzado*
Epistémico (niveles de algebrización)	EPI-E1	EPI-E2	EPI-F1	EPI-F2	EPI-M1	EPI-M2
Cognitivo (significados personales)	COG-E1	COG-E2	COG-F1	COG-F2	COG-M1	COG-M2
Instruccional (situaciones y recursos)	INS-E1	INS-E2	INS-F1	INS-F2	INS-M1	INS-M2
Contenido algebraico	ALG-E1	ALG-E2	ALG-F1	ALG-F2	ALG-M1	ALG-M2

Fuente: Godino *et al.* (2015)

Phase 2. Validation of expert judgment

Once the items were elaborated and refined, support was requested from three professors with experience in the matter who were in charge of evaluating both the viability of the items and their writing.

Phase 3. Pilot test

Table 2 shows the difficulty index in the pilot test, which was applied to 14 participants. The index [0.100] with 100 represents a very easy item, where all the students answered correctly; Likewise, the standard error of the students is represented, which allows the construction of confidence intervals of the estimates.

Tabla 2. Índice de dificultad

Momento 1: Aritmética			Momento 2: Algebra		
Ítem	*Índice de dificultad*	*Error típico*	*Ítem*	*Índice de dificultad*	*Error típico*
1	~~79.5~~	~~13.6~~	1	68.1	27.7
2	~~93.2~~	~~6.8~~	2	45.7	30.4
3	61.9	14.3	3	~~44.7~~	~~23.4~~
4	~~86~~	~~9.3~~	4	~~78.7~~	~~14.9~~
5	~~61.4~~	~~36.4~~	5	65.2	19.6
6	86.4	11.4	6	58.7	28.3
7	97.7	2.3	7	~~41.3~~	~~28.3~~
8	~~75.6~~	~~19.5~~	8	29.3	39
9	~~47.6~~	~~40.5~~	9	67.4	17.4
10	63.6	36.4			
11	56.1	17.1			
12	93.2	4.5			
13	60.5	23.3			
14	6.8	65.9			

Fuente: Elaboración propia

Phase 4. Psychometric validation

In this phase, the functioning of each of the items was ensured, as well as their discarding (as shown in Table 2). The internal consistency reliability coefficient (Cronbach's alpha) was then calculated using the SPSS software (version 25). As shown in table 3, a coefficient of 0.807 (very good reliability) was achieved, which indicates that these results can be reproduced in different samples without constituting a great bias on the part of the participants. In addition, the scores for each question were consistent with the total score of the instrument.

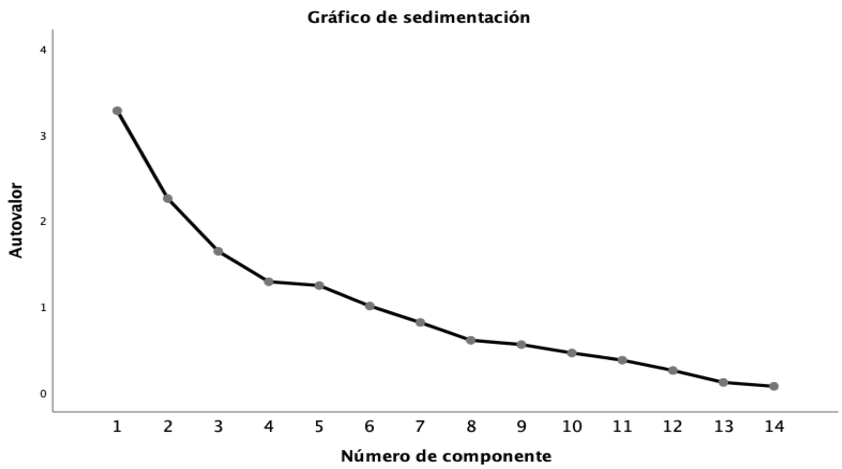
Tabla 3. Estadística de fiabilidad

Alfa de Cronbach	N.° de elementos
.807	14

Fuente: Elaboración propia

Figure 5 shows how the instrument was subjected to the unidimensionality test of the construct through the study of the sedimentation graph in the SPSS program.

Figura 5. Gráfico de sedimentación del cuestionario piloto



Fuente: Elaboración propia

Finally, Table 4 shows the percentage of variance of the first component, which was greater than 20, as well as the rest less than 10 (except 2 and 3), so it cannot be considered as a multidimensional instrument.

Tabla 4. Análisis de varianza de prueba piloto

Varianza total explicada									
	Autovalores iniciales			Sumas de cargas al cuadrado de la extracción			Sumas de cargas al cuadrado de la rotación		
Componente	Total	% de varianza	% acumulado	Total	% de varianza	% acumulado	Total	% de varianza	% acumulado
1	3.278	23.412	23.412	3.278	23.412	23.412	2.454	17.526	17.526
2	2.257	16.118	39.530	2.257	16.118	39.530	2.257	16.121	33.647
3	1.644	11.742	51.272	1.644	11.742	51.272	1.677	11.981	45.627
4	1.290	9.216	60.488	1.290	9.216	60.488	1.609	11.495	57.122
5	1.245	8.895	69.383	1.245	8.895	69.383	1.373	9.807	66.928
6	1.007	7.195	76.578	1.007	7.195	76.578	1.351	9.650	76.578
7	.816	5.829	82.407						
8	.610	4.356	86.763						
9	.560	3.998	90.762						
10	.462	3.299	94.061						
11	.379	2.704	96.765						
12	.259	1.848	98.614						
13	.120	.855	99.468						
14	.074	.532	100.000						
Método de extracción: análisis de componentes principales.									

Fuente: Elaboración propia

Conclusions

The elaboration of the instrument analyzed in this work can be a very useful tool not only to detect the previous knowledge that students have regarding differential calculus, but mainly to create didactic strategies that allow addressing these weaknesses based on the learning styles of the students. In addition, it must be anticipated that, due to the pandemic, we are living in a time where face-to-face classes have been replaced by virtual encounters, which require teaching and learning methods different from traditional ones.

For the student, therefore, this instrument can also represent an opportunity not only to know her weaknesses in the matter, but to take actions that allow to change that reality. In this regard, students must be aware of the responsibility they have in their learning processes, which has become a necessity to face the challenges that have arisen in these times of covid-19. Indeed, the pandemic showed that it is not enough just to pass the subjects, since learning is a daily task that demands a lot of independent effort, as well as, logically, the application of suitable strategies that adjust to this new reality.

Future lines of research

In the first place, the creation of various instruments that allow the teacher to diagnose the level of knowledge that their students have in a certain subject is considered a priority, since in this way strategies that meet those needs can be implemented (Rojas Maldonado, 2015). However, after developing these proposed activities, it will also be essential that investigations be carried out to determine whether they achieved the expected results.

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