Revista Iberoamericana para la Investigación y el Desarrollo Educativo
ISSN 2007 - 7467

*https://doi.org/10.23913/ride.v13i25.1347*

*Artículos científicos*

# Estudio socioeducativo de los principales errores que realizan los alumnos en el tema de la integral definida como factor que impide la competencia requerida

## *Socio-educational study of the main errors made by students in the subject of the defined integral as a factor that prevents the required competence*

## *Estudo socioeducativo dos principais erros cometidos pelos alunos na disciplina do integral definido como fator que impede a competência exigida*

**Carlos Quiroz Lima**
Instituto Tecnológico José Mario Molina Pasquel y Henríquez, Unidad Académica Puerto Vallarta, México
carlosql2702@gmail.com
https://orcid.org/0000-0001-9921-3679

**Claudio Rafael Vásquez Martínez**
Universidad de Guadalajara, México
crvasquezm@gmail.com
https://orcid.org/0000-0001-6383-270X

**Felipe Anastasio González González**
Universidad Autónoma de Tamaulipas, México
fgonzale28@hotmail.com
https://orcid.org/0000-0002-1410-8616

**Joaquín Torres Mata**
Universidad Autónoma de Tamaulipas, México
jtorresma@docentes.uat.edu.mx
https://orcid.org/0000-0002-9298-8831

Revista Iberoamericana para la Investigación y el Desarrollo Educativo
ISSN 2007 - 7467

**Irma Carolina González Sánchez**
Universidad Autónoma de Tamaulipas, México
carolinagonzalez327@gmail.com
https://orcid.org/0000-0003-2745-0178

## Resumen

En el presente trabajo se analizan algunos de los errores que realizan con más frecuencia al estudiar la integral definida los alumnos de Ingeniería en Gestión Empresarial (IGE) del Instituto Tecnológico José Mario Molina (ITJMM), así como la forma en que esto los afecta para alcanzar la competencia matemática requerida. Por ello, fue necesario investigar sobre tres elementos clave: competencia, competencia matemática y revisión en la literatura sobre los principales errores de los alumnos en el tema de la integral definida. En síntesis, al examinar los resultados de un examen diagnóstico aplicado, se observó que 89 % de los alumnos presentaban uno o más de estos tipos de problemas. Sin embargo, después de la aplicación del estudio este porcentaje se redujo a 47 %.

**Palabras clave:** errores, integral definida, socioeducativo.

## Abstract

This paper analyzes some of the main mistakes frequently made by Business Management Engineering (IGE) students at the José Mario Molina Technological Institute (ITJMM) when studying the definite integral and the way in which they affect them in order to achieve math proficiency required. For this reason, it was necessary to investigate 3 key elements: competence, mathematical competence and to review the literature on the main errors that students make on the subject of the definite integral.

When examining the results of the diagnostic test, it was observed that 89% of the students presented one or more of these types of problems. However, after the application of the study this percentage was reduced to 47%.

**Keywords:** errors, definite integral, socio-educational.

Revista Iberoamericana para la Investigación y el Desarrollo Educativo
ISSN 2007 - 7467

**Resumo**

No presente trabalho, são analisados alguns dos erros que os alunos de Engenharia em Gestão Empresarial (IGE) do Instituto Tecnológico José Mario Molina (ITJMM) cometem com mais frequência ao estudar a integral definida, bem como a forma como isso afeta para atingir a competência matemática necessária. Por isso, foi necessário investigar três elementos-chave: competência, competência matemática e revisão da literatura sobre os principais erros dos alunos na disciplina de integral definida. Em resumo, ao analisar os resultados de um teste de diagnóstico aplicado, observou-se que 89% dos alunos apresentaram um ou mais desses tipos de problemas. No entanto, após a aplicação do estudo, esse percentual foi reduzido para 47%.

**Palavras-chave:** erros, integral definida, socioeducativa.

**Fecha Recepción:** Marzo 2022 **Fecha Aceptación:** Octubre 2022

# Introduction

The students of the Engineering in Business Management (IGE) career of the José Mario Molina Technological Institute (ITJMM) when studying the topic of the definite integral make certain errors that affect not only a low percentage of approval, but also in their school practice. In this regard, Pochulu (2009) assures that the emergence of errors does not happen by chance, but due to the personal context acquired in the previous educational levels, as well as the set of instructions acquired in the educational processes.

Therefore, the elaboration of the present study arises from the need to reduce the low performance that the students of the aforementioned career present in the matter of integral calculus. The objective was to analyze some of the main errors that they frequently make on the subject of the definite integral and how this harms them in order to achieve the required mathematical competence, for which the experimental methodology was replicated, which is based on the knowledge of the teacher in Classroom (Simon, 2000).

# Objective

Analyze some of the main mistakes that ITJMM IGE students frequently make on the topic of the definite integral and how they affect them to achieve the required mathematical competence.

## Hypothesis

There are errors that ITJMM IGE students make when studying the definite integral and that act as a factor that prevents them from reaching mathematical competence.

## Research questions

What factors prevent ITJMM IGE students from achieving mathematical competence when studying the definite integral?

How do the main errors made by IGE students of the ITJMM on the topic of the definite integral affect the achievement of the required mathematical competence?

What are the main mistakes that ITJMM IGE students make on the subject of the definite integral?

# Theoretical framework

Through an investigation carried out in Chile, Espinoza et al. (2008) were able to identify four necessary competencies in the curriculum of middle and higher levels in the area of mathematics, each of which is composed in turn of a set of mathematical processes. Table 1 shows the troubleshooting.

**Table 1.** Problem resolution

| Proceso | Descripción del proceso |
|---|---|
| Entender el problema | Corresponde a la atribución de significado al enunciado, entender el contexto en el que se sitúa el problema. |
| Modelizar | Abarca los elementos de la construcción de un modelo: identificar el modelo, construir un modelo, reflexionar sobre el modelo. |
| Desarrollar y/o adaptar estrategias para resolver problemas | Corresponde a la identificación y/o construcción de una(s) estrategia(s) para abordar el problema: heurísticas, de razonamientos, casos particulares, etc. |
| Aplicar la estrategia para resolver el problema | Corresponde a la aplicación de la estrategia adoptada. |
| Interpretar la respuesta en contexto del problema | Una vez aplicada la estrategia y obtenida una respuesta, interpretar el resultado en términos del contexto del problema y responder la(s) pregunta(s) planteada en su enunciado. |
| Formular problemas | Corresponde a la formulación de un problema dadas algunas condiciones (a partir de unos datos, crear una situación problemática, etc.) |

Source: Espinoza *et al*. (2009)

The mathematical competencies for representation are mentioned in Table 2.

**Table 2.** Representation

| Procesos | Caracterización de los procesos |
|---|---|
| Entender y utilizar las relaciones entre diversas representaciones de la misma entidad. | Considera entender y utilizar diferentes representaciones que pueden darse a una misma entidad matemática (o modelo). |
| Escoger y traducir representaciones en otras. | Traducir una representación de una entidad matemática en otra representación de la misma entidad. |
| Usar representaciones para interpretar fenómenos físicos, sociales y matemáticos (construcción de modelo intermedio). | Atribuirle un significado a las representaciones y utilizarlas dentro de un contexto fenómenos físicos, sociales y matemáticos para interpretar datos. |

Source: Espinoza *et al*. (2009)

Regarding the reasoning and argumentation skills, they are shown in Table 3.

**Table 3.** Reasoning and argumentation

| Procesos | Caracterización de los procesos |
|---|---|
| Formular, investigar conjeturas matemáticas a partir de regularidades. | Formular e investigar conjeturas matemáticas que se construyen a propósito de ciertos datos provenientes de una situación intra o extra matemática. |
| Sintetizar, sistematizar y generalizar conjeturas matemáticas. | Considera la identificación de una expresión o modelo que exprese una conjetura, por ejemplo, la generalización de una propiedad matemática. También se refiere a la capacidad de sintetizar los aspectos relevantes de un tema matemático, rescatando las ideas nucleares. |
| Elegir y utilizar varios tipos de razonamiento y demostración. | Justificar y evaluar los procedimientos utilizados recurriendo a propiedades y a la lógica matemática. Frente a un mismo ente matemático utilizar distintos tipos de razonamiento para comprenderlo y/o para demostrarlo. |
| Desarrollar y evaluar argumentos. | Considera desarrollar una estructura argumentativa en el razonamiento, respecto a uno mismo o a los demás. Evalúa los elementos de un proceso de prueba: evidencia, justificaciones, demostraciones. |
| Comunicar su pensamiento matemático. | Explicar tanto de forma oral como escrita un razonamiento usado. |

Source: Espinoza *et al*. (2009)

Table 4 shows the skills for calculating and manipulating expressions.

**Table 4.** Calculation and manipulation of expressions

| Procesos | Caracterización de los procesos |
|---|---|
| Descifrar expresiones e interpretar matemáticas y/o geométricas. | Considera dar sentido a una expresión matemática o geométrica en un contexto determinado. |
| Usar y/o manipular expresiones matemáticas. | Considera la manipulación de las diferentes expresiones matemáticas, siguiendo las leyes de estructura matemática a la que pertenece. |
| Calcular y/o cuantificar. | Referido al cálculo: desarrollo de las operaciones, aplicación de una técnica, etc. |
| Comunicar la manipulación de expresiones y cálculos. | Describir de forma oral o escrita lo que se ha hecho al desarrollar un procedimiento o los cálculos llevados a cabo. |

Source: Espinoza *et al*. (2009)

# Methodology

The present work is mainly based on an experimental design that is based on the teacher's knowledge and is taken to the TDE classroom (Simon, 2000). Different design investigations have as their main objective "to observe and understand the various natural contexts that are experienced in classrooms, in order to improve the educational reality through a specific and defined instructional design" (Molina et al., 2011, p. 75).

Researchers who apply this methodology agree that in order to achieve the teaching-learning binomial, real classroom environments must be analyzed and a specific design must be developed that includes teaching strategies and tools to help raise awareness of learning and evaluation.

Similarly, design experiments emerge as an emerging dialect that seeks to support arguments built around the results of the intervention in practice, as well as the active innovation of educational contexts to glimpse the teaching and learning techniques in which the teacher is part.

For his part, Confrey (2006) defines the design of experiments as extensive educational practices that start from the curriculum, which are elaborated through a series of sequenced tasks focused on some topic, developing skills through the interaction of students between their peers and teacher.

When studying the context of the class and when carrying out the investigation, the investigation of more than one variable is required. For Molina et al. (2011) learning is an anomalous and it develops under the coercion of various factors, as well as the interaction of different characters.

Among the objectives of design research is to determine how learning is affected by variables that intervene in the class such as:

- Regarding the study of the argument of the class, it is necessary to incorporate at least one qualitative variable throughout the investigation, in such a way that it describes the development of learning by the students in the classroom and in each of the sessions. how long the investigation lasts.
- Observation of the context in the classroom on a daily basis allows us to answer questions about learning: how does it happen? Why is it due? and why does it happen?
- Collaboration between the participants and researcher within the educational environment is vital.

- The collection of data through qualitative methods allows the elaboration and direction of learning trajectories for the group.
- The presence of the researcher in the classroom throughout the study is necessary.
- The collection of data in the application of the research design allows obtaining information on the scope of learning achieved, as well as the benefits of the means used.

Within the field of mathematical didactics, the methodological paradigm of educational research is being widely used. Molina (2007) supports this statement by commenting on the disclosure of studies that were prepared with this design in various publications, as well as in research groups, such as the Design-Based Research Collective.

In works with an experimental design, the duration, space and conditions vary depending on the number of participants. Cobb et al. (2003) comment on the existence of different types of experimental design within which we can mention:

- In the design of "one by one" experiments, two small groups are formed: one for research and the other for students, whose objective is to develop a learning scale in the classroom through a sequence of teaching-learning sessions, in such a way that can be analyzed in greater depth (Cobb and Steffe, 1983).
- The design of group experiments is found in the same way. This involves a group of students who collaborate with the teacher, who may be part of the group itself (Cobb, 2000; Confrey, 2006; Confrey and Lachance, 2000; Gravemeijer, 1994).
- Teachers in training also benefit from this type of experiment, as they are supported by a research team that helps them organize and expand their knowledge (Simon, 2000).
- The design of experiments has also been applied to practicing teachers. Here professors and researchers collaborate together to develop knowledge forming a professional community (Stein *et al*., 1999).

## Analysis

The beginning of the work was carried out by the presentation of the researcher with two groups of IGE. It was highlighted that this was not the first meeting of the researcher with the students of both groups, since previously he had given them the subject of Descriptive Statistics I and II, so he knows some of their characteristics, hence they feel more comfortable confidence throughout the project.

## Development of the first session

The session was held in the ITJMM auditorium, where the two groups that would participate in the workshop course met.

52 students from the two groups of the ITJMM IGE career participated in this session; typically, both groups average 90% attendance. Subsequent sessions for group 1 were held every Thursday from 10 a.m. to 5 p.m. m. 12 noon, while for group 2 on Fridays at the same time.

At the beginning of the session, the researcher explained the dynamics of the workshop course, which would last six sessions. The purpose was to teach when a mathematical problem could or could not be solved with the use of definite integrals, for which software was used in each one's computer equipment.

## Development of the second session

This session was attended by 24 students from group 1 and 22 from group 2, so teams of 12 and 11 people, respectively, were formed. Although two hours of session were planned, with the first group it lasted five minutes.

The dynamics of the sessions took place in two phases: in the first the students worked individually and in the second in groups. In the first phase, they did it more safely because they already knew the work dynamics. When reading the exercises, doubts arose and they felt more comfortable expressing their concerns with the teacher. Sometimes the students were seen exchanging opinions with their classmates about the proposed exercises. The researcher told them to try to solve everything by applying the knowledge they had.

After individual work, the researcher asked them to find themselves in teams, preferably with the partner they chose in the first session. They were also told that they could exchange ideas and comments to answer the questions. The researcher approached each team to listen to the exchange of opinions and to answer the exercises. An example of such an exchange between the researcher (I) and the students (E) is as follows:

*(E): Can the area be calculated by approximations?*

*(I): Of course, you can use your previous knowledge of other subjects.*

*(E): Ah, so you can use geometric figures.*

*(I): Yes, as long as you remember the formula to calculate the area of the geometric figure that you remember.*

*(E): For example, the area under the curve can be approximated by a rectangle at the bottom, and by a triangle at the top.*

*(I): Very well, that would be an approximation, but the result would not be exact; now they have to think about how to be more precise in finding the area.*

## Development of the third session

At the beginning of the session, the researcher commented that a questionnaire would be carried out to identify the possible patterns of errors that students generally make when studying the subject of integral calculus. Likewise, efforts were made to detect teaching patterns and techniques to help students learn this subject. The researcher emphasized that they answered each of the questions truthfully. It is important to point out that the original questionnaire was modified due to the results obtained in the diagnostic test.

In the first question they were asked to write about the main mistakes they made when studying the topic of the definite integral. The results show that 35% accept that the subject is difficult for them due to lack of knowledge of mathematics in general; causes of distraction come in second place with 22% (eg, soccer and cell phones). Likewise, very similar are the percentages in the lack of attention and commitment (20% and 18%, respectively) due to boredom and the type of teacher.

The two main errors they make are in reasoning and calculation; about the first they say that they do not understand the concept of definite integral, and about the second that they still have algebraic problems, despite the fact that it is the preferred method for solving integrals.

Reasoning errors are due to the fact that problems or situations that motivate reasoning, argumentation and the construction of mathematical models to find their solution and interpret them properly are not put into practice in previous mathematics courses. For this reason, it is necessary that activities be incorporated in mathematics courses that include the development of processes for the development of content seen from a perspective of mathematical competences.

Some of the characteristics of mathematical competences —according to Niss (2002)— are the importance of a problematic situation that gives rise to a series of reasoning, skills and actions in the classroom. That is, seek that "all" students are capable of displaying a set of attitudes, skills and knowledge related to mathematics. Likewise, types of strategies to develop mathematical tasks in a context of competences such as a context of problem solving, research activities or project work

As for calculation errors —and despite the mechanical, algorithmic and rote use of the definition of the definite integral—, they fail to establish a connection between algebraic, geometric and analytical thinking. They have problems interpreting graphs of areas under curves and associate the concept of integral only with that of area, but isolated from other contexts, which reflects difficulties in applying the properties of the definite integral.

Other studies have shown the predominance of the algebraic mode over the graph that students have when solving integral calculus tasks. In fact, there is a dominance of algorithmic procedures versus conceptual aspects. In this regard, Giménez and Camacho Machín., 2003, p. 142). show that there is "a relatively good level in the manipulation of the algebraic algorithms that appear in the calculations of primitive functions and, however, difficulties in the conceptualization of the limit processes associated with the concept of definite integral".

Regarding the academic aspects of the integral calculus course, the students state that the unit that was most difficult for them to understand was the one related to the applications of the integral, followed by the topic of definite integrals and integration methods.

When asking the students about why it is difficult for them to solve problems that include the application of the integral, they stated that they are mostly calculus and different formulas are applied (for example, obtaining volumes). In this sense, they indicate that their teachers only use their own notes as learning strategies and sometimes use books as teaching material.

Lastly, and despite the fact that some of the workshop students had already taken the integral calculus subject, they pointed out that they were not clear about the conceptual contents of the subject, as well as the concept of the definite integral.

## Development of the fourth session

Following the planned objectives for this session, the examples of the mathematical design planned from the beginning of this work were taught, only the auxiliary graphs were added to these to better see the area to be calculated under the function. This was due to the results shown by the students in the previous sessions in that it was difficult for them to visualize the concepts of the integral and the lack of technological tools or didactic material for their learning.

Then the example planned for the session was shown, which occurred without major setbacks, since the students already mastered the basic concepts of the software and graphed the functions indicated by the problem that was shown to them through the projector. As they replicated the planned problem in their teams, they all asked and answered about how the image could be improved or the domain of the function modified. With this, the aim was for students to develop the following mathematical skills: problem solving, representation, reasoning and argumentation, calculation and manipulation of expressions.

## Development of the fifth session

Throughout the fifth session, the application of the didactic proposal was taught through examples. To do this, continuous and discrete variables were identified, which favored thinking and reasoning skills, as well as teamwork, which happened throughout the programmed examples.

The mathematical design was presented to the students through slides. They were told that these contemplate a set of properties that comprise the idea of definite integral. Likewise, it was explained to them that upon attaining design skills and applying them to a problem, they could identify whether or not it had a solution by applying the definite integral. The proposal made by Hernández (2000) for the characterization of the problems that can be solved by using the definite integral is the following.

Identification of exercises that can be solved by applying the definite integral.

Be $Q$ a set of problems that share the same characteristics, P the set of answers to the mentioned problems and $F$ a set of functions in the reals defined at each of the points that make up the interval $[a, b]$ , where for each problem $q \in Q$ your solution $p \in P$ is related to a function $f \in F$ in $[a, b]$, that is, to $\int_a^b f(x)\,dx$ provided that the following properties are met:

a) The function f is defined at every point $[a, b]$ being continuous in $[a, b]$ .
b) For this type of problem, if a constant function is associated in $[a, b]$ ; say a function h such that $h(x) = c$ for all $x \in [a, b]$ , then the solution would be $s = c(b - a)$.
c) The solution p of a problem with these characteristics, on the interval $[a, b]$, does not change if different partitions are performed on the interval $[a, b]$, so the solution of the problem is the sum of the solutions of the problem in each of the subintervals in which the interval has been partitioned $[a, b]$.

d) For this class of problems, the larger the image of the associated function, the larger the solution p of the problem.

## Development of the sixth session

At the beginning of the session, the researcher pointed out the importance of getting involved in the day's activity to determine the efficiency of the mathematical design in definite integrals. They were also reminded that in previous sessions they had been shown the procedure to follow in mathematical design, as well as the use of the software to determine when the solution to an exercise could be found by applying a definite integral, identifying continuous or discrete variables. It was explained to them that if the problem fulfilled all the requirements, then they had to characterize the integral using Reimann sums (Leithold, 1998). Likewise, they were told that the activity consisted of five questions that they had to develop with the previously mentioned and that in principle they would work individually and later in a team with the same person who did it in the previous sessions.

Then, the researcher gave them the questionnaire and they were asked if they were clear about what they were going to do. Several students answered no, so the way of working was explained to them again. Once their doubts were clarified, they began to work individually during the 30 minutes that the first stage lasted; At the end of the time, they were asked if they had any questions related to what was asked of them in each exercise.

Subsequently, the teacher began the second stage (collaborative work) and indicated that it was necessary to follow the following instructions:

- Respect participants' ideas and suggestions no matter how illogical they may seem to you; many times they have the idea of how to solve the problem, but they have a hard time expressing it.
- It was explained that collaborative work served to rely on the partner, so they could express and listen to the ideas around the proposed problem. Thus, a more complete and complex answer could be achieved than the one suggested by a single person.
- Work in teams of two people, giving priority to the teams formed in the first session. If the participants of the original team were not found, then a new one could be formed.

Exposition of a question with the procedure by work team.

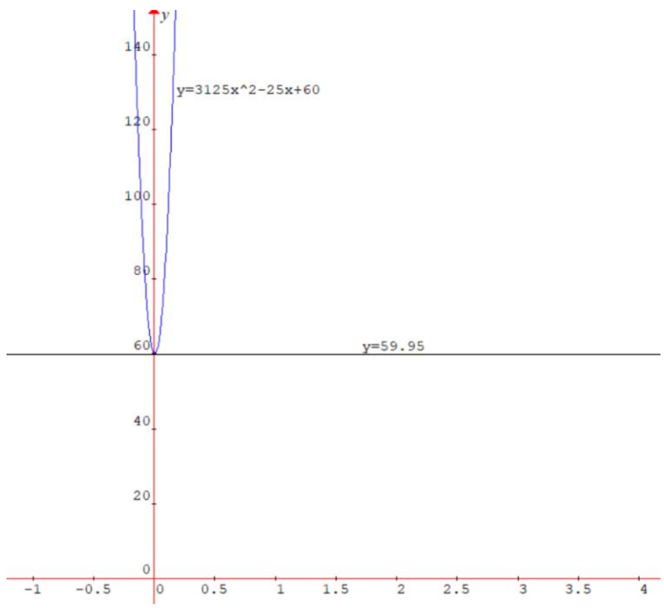
1. The average speed of a bicycle in a period of 4 hours is equal to the minimum speed of a car in a similar time interval. While the speed of the car measured in kilometers per hour

is measured by the function $f(x) = 3{,}125x^2 - 25x + 60$, being x the hours that were passing, what was the distance covered by the bicycle during the 4 hours?

Observation:

a) In this case, it is important to first identify the function that you want to know if it can be integrated under the proposed method, and since the average speed of the cyclist is equivalent to the minimum speed of the vehicle, the minimum point of the function must be obtained. function of the vehicle, which is $x = 0.004$ as $f(0.004) = 59.95$ Thus, the cyclist's function is $f(x) = 59.95$ both graphs are shown in figure 1 in an interval [0,4]

**Figure 1.** Graphs of the functions $f(x) = 3{,}125x^2 - 25x + 60;\ f(x) = 59.95$




Fuente: Elaboración propia

b) Since the function associated with the cyclist is constant, it is possible to obtain the distance covered in the interval [0, 4]. Namely:

$$S[0,4]$$

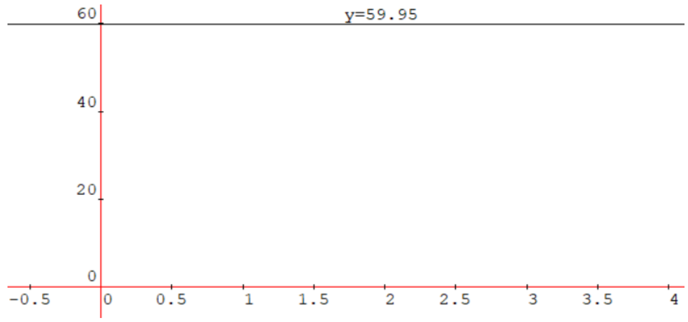
$$f(4)(4-0)$$

$$59.95 * 4$$

$$239.8$$

As shown in figure 2.

**Figure 2.** Graph of the function $f(x) = 59.95$

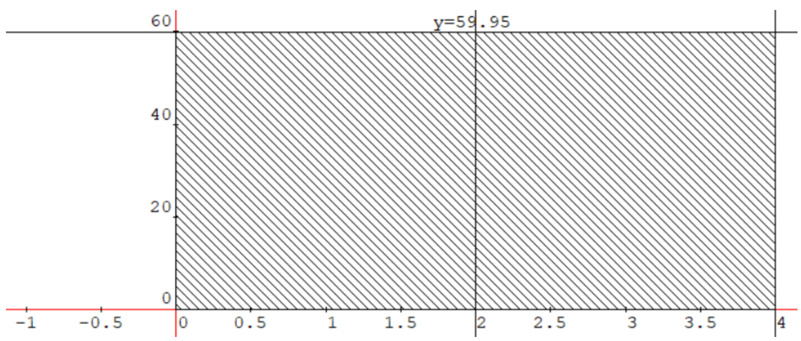



Fuente: Elaboración propia

c) On the other hand, the larger the area obtained by the function, the greater the solution of the problem.

d) Now, if partitions of the interval are obtained [0, 4], let's say in [0, 2] y [2, 4], and the distance traveled by the cyclist in each of the subintervals of the partition is calculated, the sum of the distances traveled in each of the subintervals will always be equal to the total increase in the interval [0, 4] as shown in figure 3.

**Figure 3.** Graph of the function $f(x) = 59.95$ with $n = 2$




Source: self made

Namely:

$$S[0,2] + S[2,4]$$
$$f(2)(2-0) + f(4)(4-2)$$
$$59.95(2) + 59.95(2)$$
$$119.9 + 119.9 = 239.8$$

which is in the solution.

# Results

The research focused on two questions:

1. What were the main mistakes that students made in the topic of definite integral? For this, a sequence of six work sessions in the classroom was designed and applied:
   - In order to build new knowledge, it was necessary to return to previous knowledge, associating problem solving with an argument and representation of them.
   - The resolution of the proposed mathematical problems should have a practical approach and immediate application.
   - Priority was given to collaborative work, developing the sessions in a natural context that would help train future ITJMM engineers in business management.
2. What were the main errors that prevented ITJMM IGE students from achieving the required mathematical skills in the subject of the definite integral? In this case, efforts were prioritized in two areas of contribution:
   - The use and application of the TDE methodology with emphasis on the achievement of the required mathematical skills.
   - Teamwork to develop the cognitive skills required in the activities.

## First question: what were the main mistakes that the IGE students of the ITJMM made in the topic of definite integral?

This question was analyzed in two sessions: the first at the time of applying the diagnostic test to define the procedure to be followed. In the second session, a survey was used so that the students pointed out the main errors when studying the topic of the definite integral.

Both instruments served to identify three types of errors related to the calculation, the problem-solving process and the application of inadequate algorithms. The results of the diagnostic test indicated that 89% of the students presented one or more of these types of problems. However, after the use of the didactic tool, this percentage was reduced to 47%.

In the case of errors made during the process of solving a problem, it was found —before the application of the instrument— that they presented algebraic difficulties (for example, they did not distinguish when a function was discontinuous in an interval) and they did not know if the variable that is involved in a problem is of the discrete or continuous type.

The data shows that 64% of the students make errors throughout the solution process, while the remaining 36% are more consistent with the calculation procedures.

At the end of the application of the instrument and after insisting on the identification of the type of variable involved in the problems and the algebraic care, the percentage decreased to 42%. In this case, it was also found that students make this type of error mainly due to lack of mathematical maturity, and lack of reading and understanding of problems.

Very similar is the response for ignorance or application of the rules. Before the application of the instrument, only 21% of the students were able to respond satisfactorily to questions 2 and 3 of the diagnostic test. As an exercise and throughout the practice process, the same problems were solved by the students in questions 5 and 6, respectively. On this occasion, 69% of the students answered the questions correctly.

The application of the instrument caused great interest and motivation in the students in the classroom, which was evidenced in the participation and constant formation of questions. Its application arises from the need for students to identify mathematical content and concepts in the search for solutions to real problems and, after analysis, to be able to affirm whether or not the problem can be modeled using the definite integral.

Although the percentages of errors made by the students decreased with the application of the proposed material, they are still high. It would be risky, therefore, to think that with this didactic material or another the percentage of errors would reach zero. However, it could decrease if there were the opportunity to use more hours in the application of this material.

## Second question: what factors prevent ITJMM IGE students from achieving the required mathematical skills in the subject of the definite integral?

To answer this question, two sources of contribution were determined:

- The format of the TDE methodology designed to achieve the required skills.
- The effort involved in teamwork for the development of cognitive activities.

The elaboration of this work is described through the effort of the teams in search of the learning expectations organized in six sessions in the classroom and linking the work objectives with the mathematical competences. In this way, the assumptions that were planned in the elaboration of the activities and the active participation shown by the students throughout the project were contrasted.

It was noted that, in particular, the context of certain activities could influence the goals of achieving one or more competencies; however, in practice this did not occur. That is, the competence was not reached or the indicators were not consistent with the complexity of the activities, which means that although an exercise has been classified as reflection, the IGE students of the ITJMM worked on it only as one more exercise.

In summary, the procedure to achieve the required mathematical skills allowed obtaining information on the contribution of the study to the achievement of the desired expectations in learning. It is recalled that the learning expectations, the objectives and the competences of each proposed exercise were elaborated taking into account the resolutions of the results obtained in a collaborative way. Therefore, the achievement of the competence of each of the students or that any of them will show an advance or be promoted is not guaranteed. In fact, it is not mentioned that all the members that made up the teams have achieved such an objective.

The contribution presented in this study shows a scenario of the formation of work teams that give evidence of the proposed learning expectations. Regarding the question about the promotion of mathematical competence by the student, it is considered that this will only be achieved if the previous knowledge acquired is taken into account. In this sense, the best context for achieving these skills is the classroom.

Mathematical competence promotes the use of awareness generating and managing skills and knowledge from a perspective of personal competence that includes cooperation, management and application of learning strategies and self-assessment during learning.

For the promotion of mathematical competence, it is necessary to develop and design activities that include numerical exercises for immediate application in real and functional contexts. On the other hand, it is considered that in order for students to achieve the required mathematical maturity, it is necessary to reflect on the course that higher education must take in the training of students so that they are mathematically competent.

## Discussion

Various authors have worked in the search to identify the difficulties and errors that students have in learning mathematics. Rico (1995) states that in the nineties these were characterized by the organization of mathematics curricula and objectives, as well as certain currents of psychology and pedagogy. The investigations of Brousseau et al. (1986) describe that the errors made by the students show a consistent pattern in the use of wrong procedures. The use of problem lessons is used to uncover misconceptions and thus build discussions in order to find a resolution (Bell, 1986). Borassi (1987) and other authors use the mistakes made by the student as a motivational instrument for the development of activities where problems are raised and their resolution.

Other studies concerning error analysis focus on the algebraic language. For example, Booth (1984) states that many of these errors are attributed to aspects such as the transition from arithmetic to algebra and the nature of the answers in the understanding of arithmetic and the inappropriate use of formulas or procedural rules. In Ruano et al. (2008) analyzes the errors that students make in the operational, structural and procedural aspects when they work on problematic situations that involve some of the formal substitution, generalization and modeling processes. On the other hand, research has also been carried out that deals with the origin and cause of errors from the point of view of the difficulties inherent in mathematics, as well as the study of the difficulties that lead to the teaching-learning process (Socas, 2007).

Collaborative work was also part of this work. In this sense, Pérez (2007) comments that it is a social process where there is not only interaction between students, but also between student and teacher. For this reason, Coll and Sánchez (2008) discuss basic aspects to be taken into account in the development of models for the analysis of interaction and educational practice in the classroom.

As for the errors made during the process of solving a problem, two very common ones were detected: algebraic errors (for example, not distinguishing when a function was discontinuous in an interval) and not knowing if the variable being involved in a problem was of the discrete or continuous type. The first is more distinguished in algebraic type problems and the second can be seen in the solution of real problems, where a definite integral is required or not.

## Conclusions

The methodology used in this work served not only to know the actions of the students in the subject of the definite integral, but also to be able to guide them in their learning process, which was simplified thanks to the fact that the researcher remained in the classroom from the beginning. first day of class and shared with the students the work of the subject. The participation of the teacher in each of the sessions allowed him to learn more about the stages of work, which was useful to prepare more precise explanations due to the actions manifested by the students on the influences demonstrated.

In summary, it has been positive to work with two groups of IGE students from the ITJMM. Indeed, the application of this methodology allowed the researcher to design and apply each of the sessions in order to offer a greater benefit in student learning.

## Future lines of research

The following lines of research emerge from this study:

- Studies in high school and university classrooms on the errors that students make in integral calculus.
- Local studies in educational centers on the effects of the pandemic and the post-pandemic period where students make errors about integral calculation.

## References

Bell, A. (1986). Enseñanza por diagnóstico. Algunos problemas sobre números enteros. Enseñanza de las Ciencias, 4(3), 199-208.

Boothe, L. (1984). Algebra: Children's Strategies and Errors: NFER-Nelson. BoothAlgebra: Children's Strategies and Errors1984.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. For the learning of Mathematics, 7(3), 2-8.

Brousseau, G., Davis, R. B., & Werner, T. (1986). Observing students at work. In Perspectives on mathematics education (pp. 205-241). Springer.

Cobb, P. (2000). Conducting teaching experiments in collaboration with teachers.

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. Educational researcher, 32(1), 9-13.

Cobb, P., & Steffe, L. P. (1983). The constructivist researcher as teacher and model builder. Journal for research in mathematics education, 14(2), 83-94.

Coll Salvador, C., & Sánchez Miguel, E. (2008). Presentación: El análisis de la interacción alumno-profesor: líneas de investigación. Revista de educación.

Confrey, J. (2006). The evolution of design studies as methodology. na.

Confrey, J., & Lachance, A. (2000). Transformative Teaching Experiments through Conjecture-Driven Research Design.

Espinoza, L. (2008). Análisis de las competencias matemáticas en NB1. Caracterización de los niveles de complejidad de las tareas matemáticas.

Giménez, C. A., & Machın, M. C. (2003). Sobre la investigación en didáctica del análisis matemático. Edición Especial: Educación Matemática, 135.

Gravemeijer, K. (1994). Educational development and developmental research in mathematics education. Journal for research in Mathematics Education, 25(5), 443-471.

Hernández, R. (2000). Propuesta didáctica para identificar y resolver los problemas que requieren del cálculo de una integral definida o de la derivada de una función real en un punto Tesis de Doctorado en Ciencias Pedagógicas, Universidad de La Habana, Ciudad …].

Leithold, L. (1998). El cálculo (Vol. 343). Oxford University Press México.

Molina González, M. (2007). Desarrollo de pensamiento relacional y comprensión del signo igual por alumnos de tercero de educación primaria.

Molina, M., Castro, E., Molina, J. L., & Castro, E. (2011). Un acercamiento a la investigación de diseño a través de los experimentos de enseñanza. Enseñanza de las ciencias: revista de investigación y experiencias didácticas, 75-88.

Niss, M. (2002). Mathematical competencies and the learning of mathematics. The Danish.

Pochulu, M. (2009). Análisis y categorización de errores en el aprendizaje de la matemática en alumnos que ingresan a la universidad. Colección Digital Eudoxus(8).

Pérez, M. M. (2007). El trabajo colaborativo en el aula universitaria. Laurus, 13(23), 263-278.

Rico, L. (1995). Errores y dificultades en el aprendizaje de las matemáticas.

Ruano Barrera, R. M., Socas Robayna, M. M., & Palarea Medina, M. d. l. M. (2008). Análisis y clasificación de errores cometidos por alumnos de secundaria en los procesos de sustitución formal, generalización y modelización en álgebra. PNA.

Simon, M. (2000). Research on mathematics teacher development: The teacher development experiment. In Handbook of research design in mathematics and science education (pp. 335-359). Lawrence Erlbaum Associates Publishers.

Socas, M. (2007). Dificultades y errores en el aprendizaje de las matemáticas. Análisis desde el enfoque lógico semiótico.

Stein, M. K., Smith, M. S., & Silver, E. (1999). The development of professional developers: Learning to assist teachers in new settings in new ways. Harvard educational review, 69(3), 237-270.

| Rol de contribución | Autor(es) |
|---|---|
| Conceptualización | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Metodología | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Software | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Validación | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Análisis formal | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Investigación | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Recursos | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Curación de datos | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |

| | |
|---|---|
| Escritura y preparación del borrador original | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Escritura, revisión y edición | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Visualización | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Supervisión | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Administración de proyectos | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |
| Adquisición de fondos | Carlos Quiroz Lima (Principal), Claudio Rafael Vásquez Martínez (Principal), Felipe Anastasio González González (Igual), Joaquín Torres Mata (Igual), Irma Carolina González Sánchez (Igual). |