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Artículos científicos

Propuesta de actualización de saberes en campos numéricos para estudiantes de pedagogía en telesecundaria

***Proposal for Updating Knowledge in Numerical Fields for Students of
Pedagogy in Telesecundaria***

***Proposta de atualização de conhecimentos em campos numéricos para
estudantes de pedagogia em telesecundaria***

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Resumen

El presente artículo muestra una secuencia didáctica dirigida a los estudiantes de la licenciatura en Enseñanza y Aprendizaje en Telesecundaria de la Escuela Normal Urbana Federal Prof. Rafael Ramírez de Chilpancingo, Guerrero, con la finalidad de sistematizar sus conocimientos en los diversos campos numéricos. El objetivo es conocer y manejar la clasificación de los números reales, porque es de suma importancia para los docentes en formación: si saben las características de cada uno de ellos, podrán realizar las operaciones adecuadas, sin confundirse entre los diversos campos numéricos. El presente trabajo se fundamenta en la teoría de David Ausubel (Méndez, 1993), que trata sobre el aprendizaje significativo. Por otra parte, es muy común que los estudiantes de secundaria confundan las reglas de operación que se aplican en la suma y resta, y las utilicen para multiplicar y dividir; esto tiene una explicación, los conocimientos matemáticos son como una espiral, cada vez se van incrementando y las reglas van cambiando, por eso cada campo numérico nuevo



representa para el alumno un conflicto cognitivo, de tal manera que no es la misma regla sumar dos números positivos que sumar dos fracciones que tienen diferente denominador. El resultado de la suma de dos números enteros positivos también es positivo y es mayor que cualquier sumando; mientras que la suma de las fracciones tiene otro procedimiento muy distinto; en este caso, se ocupó el mínimo común múltiplo y las fracciones equivalentes para calcular la suma. Lo mismo sucede con los demás campos numéricos.

Palabras clave: actualización docente, números reales, operaciones básicas, secuencia didáctica, telesecundaria.

Abstract

The present article shows a didactic sequence directed to the students of the degree in Teaching and Learning in Telesecundaria of the Escuela Normal Urbana Federal Prof. Rafael Ramírez of the city of Chilpancingo, Guerrero, with the purpose of systematizing their knowledge in the various numerical fields. The objective is to know and manage the classification of real numbers, because it is of utmost importance for teachers in training: if they know the characteristics of each one, they will be able to carry out the appropriate operations, without being confused between the various numerical fields. The present work is based on the theory of David Ausubel (Méndez, 1993), which deals with meaningful learning. Moreover, it is very common for high school students to confuse the rules of operation that apply to addition and subtraction, and use them to multiply and divide; this has an explanation, mathematical knowledge is like a spiral: each time it increases and the rules change, so each new number field represents a cognitive conflict for the student, in such a way that adding two numbers is not the same rule positives that add two fractions that have a different denominator. The result of the sum of two positive integers is also positive and is greater than any addend; while the sum of the fractions has another, very different procedure; in this case, the least common multiple and the equivalent fractions were used to calculate the sum. The same is true for the other number fields.

Keywords: teaching update, real numbers, basic operations, didactic sequence, telesecundaria.



Resumo

Este artigo apresenta uma sequência didática direcionada aos alunos da graduação em Ensino e Aprendizagem em Telesecundaria da Escuela Normal Urbana Federal, Prof. Rafael Ramírez de Chilpancingo, Guerrero, com o objetivo de sistematizar seus conhecimentos nas diversas áreas numéricas. O objetivo é conhecer e gerenciar a classificação de números reais, pois é de extrema importância para os professores em formação: se eles conhecerem as características de cada um deles, serão capazes de realizar as operações apropriadas, sem serem confundidos entre os vários campos numéricos. O presente trabalho é baseado na teoria de David Ausubel (Méndez, 1993), que trata da aprendizagem significativa. Por outro lado, é muito comum que estudantes do ensino médio confundam as regras de operação que se aplicam à adição e subtração e as usam para multiplicar e dividir; isso tem uma explicação, o conhecimento matemático é como uma espiral, cada vez que eles aumentam e as regras mudam; portanto, cada novo campo numérico representa um conflito cognitivo para o aluno, de forma que a adição de dois números não é a mesma regra positivos que adicionam duas frações que têm um denominador diferente. O resultado da soma de dois números inteiros positivos também é positivo e é maior que qualquer adendo; enquanto a soma das frações tem outro procedimento muito diferente; neste caso, o mínimo múltiplo comum e as frações equivalentes foram usadas para calcular a soma. O mesmo vale para os outros campos numéricos.

Palavras-chave: atualização de ensino, números reais, operações básicas, sequência didática, telesecundaria.

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Introduction

All teachers who teach the subject of mathematics have the obligation to know the classification of real numbers in order to properly perform the operations, because each numerical field has its own rules of operation.

In real terms, we find that newly admitted students to the Telesecundaria Telesecundaria Teaching and Learning degree at the Prof. Rafael Ramírez Federal Urban Normal School of Chilpancingo, Guerrero, Mexico, have low levels of mathematical knowledge. This represents a challenge to train good teachers, which is why documentary and field work is carried out that allow normalista teachers to correct these irregularities in the students.

Therefore, a didactic sequence was designed that allowed students to collectively and individually analyze numerical problems, make reflections and conclusions for a better understanding.

Collective work allows knowing the different points of view of other colleagues to solve problems. The ideas may or may not be good, that is why they are analyzed collectively, to institutionalize knowledge, all this is done under the supervision of the teacher, who has the role of mediator.

This article describes the evolution of numeric fields. On some occasions the video of Sáenz (2019) is paraphrased, but the history of mathematics is also taken into account, as proposed by Struik (1998). It begins with the natural numbers, which were the first to be created by humanity, then with the integers, moving on to the rational numbers and ending with the irrational ones. The passage of each one of them is seen with the difficulty of an operation that cannot be solved with the numerical field analyzed up to that moment; In addition, they are presented as a set and on a number line. The latter is intended to make what is built more explicit.

Methodology

The development of this work was carried out as follows:

- a) The didactic sequence of real numbers was designed.
- b) The set of natural numbers was presented with some operations that cannot give results in that numerical field.



- c) In the same way it was done with the other numbers: integers, rational and irrational.
- d) In each field the operating rules were analyzed and the corresponding exercises were carried out, to become familiar and build their own definitions.

Development of activities

- Write the numbers you know or have occupied:
- How are they called?

The mathematical community has established a classification of the numbers that are presented below with some reinforcement and reflection activities. The collective and individual conclusions are important to be made at the end of each activity, as well as at the end of the numerical field analyzed. It is important to agree with the group to institutionalize the knowledge that is being learned.

We started!

The classification of real numbers

Natural numbers

Figura 1. Los números en los pueblos primitivos



Fuente: Baldor (1982)

Historically, man has counted out of necessity. His records are in various materials. One of the oldest is the Lebombo bone (they are baboon bones), according to Sánchez (February 2, 2014), which has 28 marks, so it can be related to the lunar or menstrual cycle.

Figura 2. Hueso de Lebombo

Fuente: Sánchez (2 de febrero de 2014)

Figura 3. Babuinos

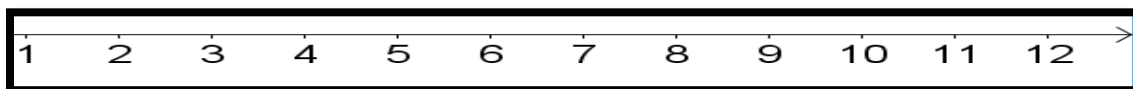
Fuente: Wikipedia (26 de noviembre de 2019)

The Lebombo bone is an instrument to count the time of some natural cycle; It is 35,000 years old and was found in Border Cave, in the Lebombo Mountains, between South Africa and Swaziland (Sánchez, February 2, 2014).

According to the MateFacil channel (August 8, 2016), the first numbers are called natural numbers and are represented by a capital N with a double line (\mathbb{N}). Man, when counting, counted what he saw in nature, what he did not see he did not count, which is why the invention of zero was difficult for many cultures. From this perspective, the first number is one and not zero.

On a number line it is represented as follows (see figure 4).

Figura 4. Números naturales



Fuente: Elaboración propia

With natural numbers you can do various arithmetic operations, such as addition, subtraction, multiplication and division.

Tabla 1. Sumas de números Naturales

Realiza las siguientes sumas
a) $23 + 45 =$
b) $34 + 78 =$
c) $103 + 59 =$

Fuente: Elaboración propia

Answer the following questions

- Are all the results natural numbers?
- Are other types of numbers outside the natural ones required?

Tabla 2. Restas de números Naturales

Realiza las siguientes restas
a) $150 - 60 =$
b) $20 - 20 =$
c) $54 - 100 =$

Fuente: Elaboración propia

Answer the following questions

- Are all the results natural numbers?
- So other types of numbers outside the natural ones are required?

As society developed, the first number field was insufficient to solve problems, so it was necessary to create another number field.

Tabla 3. Multiplicación de números Naturales

Realiza las siguientes multiplicaciones
a) $15 \times 8 =$
b) $23 \times 12 =$
c) $189 \times 45 =$

Fuente: Elaboración propia

Answer the following questions

- Are all the results natural numbers?
- Are other types of numbers outside the natural ones required?

Make the following divisions

$$a) \frac{15}{3} =$$

$$b) \frac{60}{5} =$$

$$c) \frac{25}{6} =$$

Answer the following questions

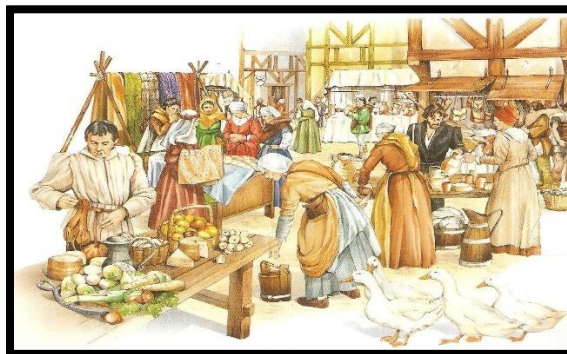
- Are all the results natural numbers?
- Are other types of numbers outside the natural ones required?

Answering the questions, we realize that the results of the addition are always natural numbers. For this operation there are no problems to create another numeric field; However, for the subtraction operations, there is an obstacle to solve: if the first number (minuend) is less than the second number (subtrahend), the result (difference) is not located within the natural numbers, so Therefore, it was essential to create another numeric field.

In the case of multiplication operations, there is no problem, all results are natural numbers; but in the case of division, there are certain results that do not belong to the field of natural numbers, for example: $\frac{1}{3}$, $\frac{-2}{8}$, etc.

Whole numbers

Figura 5. Ferias medievales



Fuente: <http://www.escuelapedia.com/ferias-medievales/>

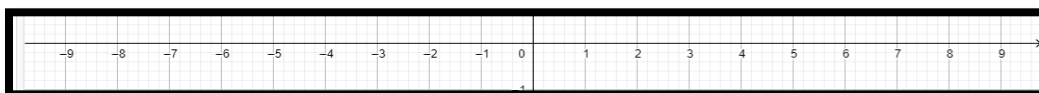
Following Wikipedia (July 19, 2020), when trade appears in society, it became necessary to numerically represent various situations, for example: when all the merchandise had been sold (zero), the profits from a certain sale (numbers positive) and losses obtained for various reasons (negative numbers). This originated the numerical field of integers, which are represented by the symbol \mathbb{Z} , and mathematically is written as follows:

$$\mathbb{Z} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}$$

The three dots mean 'and so on'.

The whole numbers represented on the number line are shown in Figure 6.

Figura 6. La recta numérica



Fuente: Elaboración propia

With the creation of this numeric field, the problem of subtraction that had been raised previously is solved: $54 - 100 = -46$. In practical terms, the problem could have been stated in the following way: "Mr. Carlos had \$ 54.00 and he paid that amount to a debt of \$ 100.00, how much was owed?" The answer is \$ 46.00, the minus sign is placed before the number to indicate that it does not correspond to a positive number.

This new numerical field has other rules that we will analyze with their respective operations, but we need prior knowledge that will contribute to its understanding, so the absolute value will be studied first.

Absolute value

The absolute value of an integer is the natural number that results from suppressing its sign. The absolute value can be understood as the distance between the given number and zero. We will write the absolute value between vertical bars.

Tabla 4. Simbología y lectura de los números absolutos

Simbología	Se lee como:
$ -5 = 5$	“El valor absoluto de -5, es igual a 5”
$ 5 = 5$	“El valor absoluto de 5, es igual a 5”
$\left -\frac{2}{3}\right = \frac{2}{3}$	“El valor absoluto de $-\frac{2}{3}$, es igual a $\frac{2}{3}$ ”

Fuente: Elaboración propia

Determine the absolute value of the following numbers

Tabla 5. Números absolutos

a) $ -8 =$	b) $ -12 =$	c) $ 6 =$	d) $ 46 =$
e) $\left \frac{-2}{8}\right =$	f) $\left \frac{-7}{9}\right =$	g) $\left \frac{4}{5}\right =$	h) $ 0.5 =$

Fuente: Elaboración propia

Operations with whole numbers

In the sum of two numbers that have equal signs:

- If it is a positive number, it will move to the right.
- If it is negative, it will shift to the left.

Solve the following sums

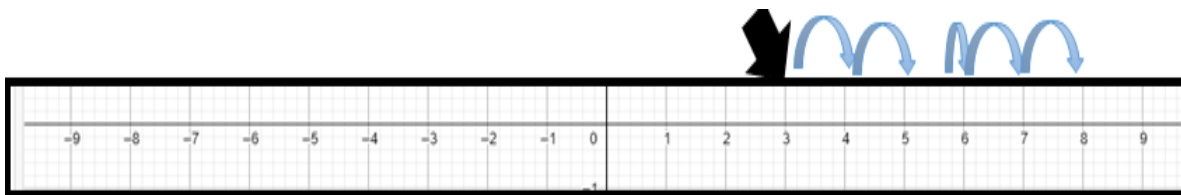
a) $3 + 5 =$



Process:

- 1) The number three is located on the number line
- 2) It shifts five units to the right because five is positive.

Figura 7. Suma de dos números positivos



Fuente: Elaboración propia

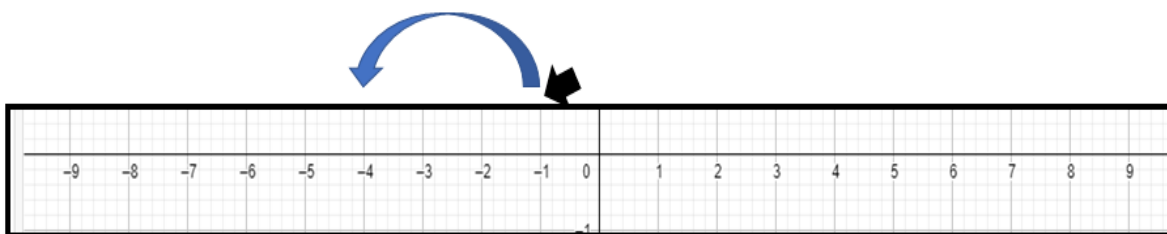
The result is +8

b) $-1 - 3 =$

Process:

- 1) The number -1 is located on the number line.
- 2) Shift three units to the left because three is negative.

Figura 8. Suma de dos números negativos



Fuente: Elaboración propia

The result is - 4.

Reflection: if we add (simplify to a single number) two numbers with equal signs, the operation is a sum. The result will have the sign of the addends (positive or negative), as the case may be.

Tabla 6. Suma de números con igual signo

Caso I: Cuando ambos números tienen el mismo signo, la operación es una suma; el signo es positivo o negativo según sea el caso.			
Ejemplos:			
a) $2 + 3 = +5$	b) $7 + 2 = +9$	c) $4 + 6 = +10$	d) $12 + 14 = +26$
e) $-2 - 3 = -5$	f) $-7 - 2 = -9$	g) $-4 - 6 = -10$	h) $-12 - 14 = -26$
Resuelve las siguientes sumas en tu cuaderno			
a) $6 + 3 =$	b) $12 + 5 =$	c) $2 + 9 =$	d) $12 + 8 =$
e) $15 + 7 =$	f) $17 + 5 =$	g) $11 + 3 =$	h) $14 + 2 =$
i) $24 + 11 =$	j) $17 + 12 =$	k) $27 + 18 =$	l) $32 + 45 =$
m) $-2 - 8 =$	n) $-12 - 6 =$	ñ) $-11 - 10 =$	o) $-7 - 9 =$
p) $-12 - 25 =$	q) $-8 - 15 =$	r) $-9 - 12 =$	s) $-22 - 11 =$
t) $-17 - 6 =$	u) $-9 - 11 =$	v) $-27 - 17 =$	w) $-42 - 35 =$

Fuente. Elaboración propia

In the sum of two numbers that have different sign:

- If it is a positive number, it will move to the right.
- If it is negative, it will shift to the left.

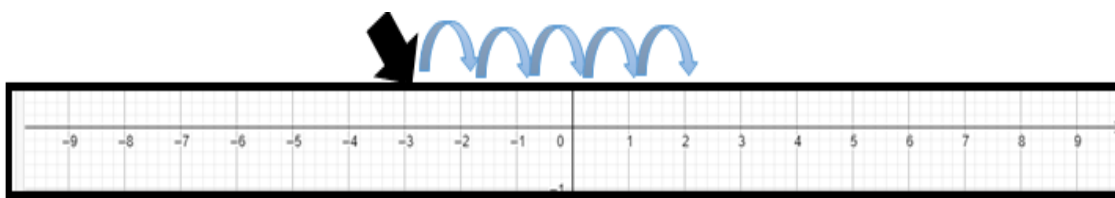
Solve the following sums

a) $-3 + 5 =$

Process:

- 1) The number -3 is located on the number line
- 2) It shifts five units to the right because five is positive.

Figura 9. Suma de dos números de diferente signo



Fuente: Elaboración propia

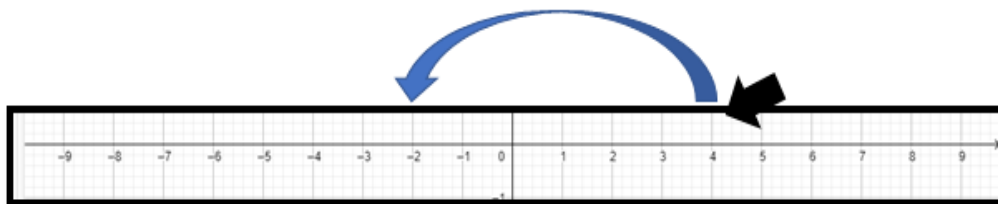
The result is + 2

b) $4 - 6 =$

Process:

- 1) The number four is located on the number line.
- 2) It shifts six units to the left because six is negative.

Figura 10. Cuando el número negativo es mayor que el positivo



Fuente: Elaboración propia

The result is - 2

Reflection: if we add (simplify to a single number) two numbers with different signs, the operation is a subtraction. Taking the numbers with absolute value, the smallest is subtracted from the largest number, and the sign of the largest remains.

The following exercises can help you better understand the operations (see table 4).

Tabla 7. Suma de números con diferente signo

Caso II: Cuando los números tienen diferente signo, la operación es una resta, es decir, al número de mayor absoluto se le resta el de menor valor absoluto, quedando el signo en el resultado el de mayor valor absoluto.			
Ejemplos:			
a) $2 - 6 = - 4$	b) $4 - 10 = - 6$	c) $6 - 16 = - 10$	d) $8 - 12 = - 4$
e) $- 25 + 10 = - 15$	f) $- 17 + 27 = 10$	g) $- 22 + 34 = +12$	h) $- 23 + 15 = - 8$
Resuelve las siguientes operaciones en tu cuaderno			
a) $5 - 9 =$	b) $3 - 12 =$	c) $7 - 15 =$	d) $9 - 17 =$
e) $12 - 23 =$	f) $28 - 23 =$	g) $39 - 25 =$	h) $12 - 32 =$
i) $- 19 + 25 =$	j) $- 31 + 25 =$	k) $- 22 + 34 =$	l) $32 - 24 =$

Fuente. Elaboración propia

Rational numbers

Figura 11. Balanza comercial



Fuente: Céspedes (27 de septiembre de 2017)

Trade and construction of works have greatly influenced the development of societies and numerical fields, for example, how long must each side of a hexagon be? If we know that its perimeter is 25 m, this operation has not yet been solved with the numerical fields seen so far ($25 \div 6$).

Therefore, it was necessary to create another numeric field. This new numeric field includes the previously known numbers and its definition is as follows: set of numbers that can be written as a fraction, where the numerator and denominator belong to the whole number field and the denominator is different from zero.

Mathematically it is written as follows:

$$\mathbb{Q} = \left\{ \frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0 \right\}$$

Rational numbers, being from another numeric field, have other rules to perform operations. Some students apply the same rules of the numerical field of the whole numbers and logically it is an error. What was previously a success in operations, now it turns out that things change and the results are given another interpretation.

Each new knowledge represents a cognitive conflict for students, which, if overcome, can acquire the following school levels (subjects with a higher degree of complexity), which increasingly require greater mathematical bases for their mastery.

Interpretation of fractions

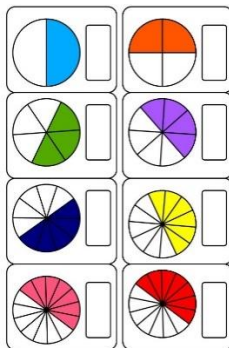
According to Your Online Teacher (May 13, 2015), common fractions are those that have a numerator and a denominator, for example:

$$\frac{3}{8} = \frac{\leftarrow \text{Numerador}}{\leftarrow \text{Denominador}} = \frac{\text{indica las partes que se toma de la unidad}}{\text{indica las partes en que se divide la unidad}}$$

Representation of fractions

Write the fraction that corresponds to the colored part

Figura 12. Juegos con fracciones: lectura, comparación, fracciones



Fuente: Rodríguez (s. f.)

Answer the following questions

- 1) What are the fractions like?
- 2) How can you check?

Fraction operations

The following activities take up the video of "Understanding the addition and subtraction of fractions" (Matemática profe Alex, 28 de octubre de 2018):

- Sum of fractions that have the same denominator:

$$a) \frac{6}{4} + \frac{7}{4} = \frac{13}{4} \quad b) \frac{5}{7} + \frac{12}{7} = \frac{17}{7} \quad c) \frac{14}{12} - \frac{6}{12} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3}$$

$$d) \frac{12}{8} - \frac{13}{8} - \frac{5}{8} = \frac{-6}{8} = -\frac{3}{4}$$

Solve the following sum of fractions

Tabla 8. Suma de fracciones homogéneas

1) $\frac{5}{3} + \frac{12}{3} =$	2) $\frac{7}{5} + \frac{8}{5} =$	3) $\frac{5}{9} + \frac{14}{9} =$	4) $\frac{4}{14} + \frac{10}{14} =$
5) $\frac{15}{7} + \frac{13}{7} =$	6) $\frac{17}{8} + \frac{18}{8} =$	7) $\frac{28}{3} + \frac{14}{3} - \frac{7}{3} =$	8) $\frac{43}{4} + \frac{10}{4} - \frac{20}{4} =$
9) $\frac{12}{7} - \frac{53}{7} =$	10) $\frac{19}{8} + \frac{14}{8} =$	11) $\frac{8}{23} + \frac{4}{23} - \frac{7}{23} =$	12) $\frac{23}{7} - \frac{10}{7} - \frac{50}{7} =$
13) $\frac{15}{6} + \frac{17}{6} =$	14) $\frac{97}{12} + \frac{18}{12} =$	15) $\frac{-8}{2} + \frac{54}{2} - \frac{7}{2} =$	16) $\frac{43}{15} + \frac{11}{15} - \frac{27}{15} =$

Fuente. Elaboración propia

- Sum of fractions that have different denominators, but the greater is a multiple of the smaller:

$$\frac{1}{2} + \frac{2}{4} =$$

In this regard, these cannot be added directly; they have to be expressed in equivalent fractions.

- 1) Observe the smallest of the denominators (in this case it is the number two), is it a divisor of the other of the other denominator? The answer is yes, then:
- 2) The lower denominator divides the larger denominator. The result (quotient) multiplies its two elements: numerator and denominator.
- 3) The fraction obtained is equivalent to $\frac{1}{2} = \frac{2}{4}$
- 4) The equivalent fractions are added, taking into account only the numerators. Already as equivalent fractions, the sum can be done as follows:

$$\frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1$$

It should be noted that in the case of subtraction it is done in a similar way.

- Sum of fractions that have different denominators, but the larger is not a multiple of the smaller:

$$\frac{1}{3} + \frac{2}{4} =$$

The following is more general and can be used for any type of common fraction, according to Mora (2004).

- 1) A cross multiplication is done in such a way that equivalent fractions are obtained.
- 2) The fractions are added:

$$\frac{1}{3} + \frac{2}{4} = \text{it is as follows: } \frac{4}{12} + \frac{6}{12} = \frac{10}{12}$$

This is also done with subtraction.

Now, how do you know if one number is divisible by another? This question is answered with the criteria of divisibility, that is, those rules that say when a certain number is divisible by another because it meets certain criteria.

Criterion of divisibility of the number two. What numbers does two divide into?

Circle the number two and indicate which numbers are divisible by that number from the table below

Tabla 9. Números divisibles por el número dos

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Fuente: Elaboración propia

- Write the numbers that are divisible by the number two:
- According to the written numbers, what numbers end in:
- If the numbers increased, do you think the behavior would be the same?
- In general terms, mention what numbers the number two divides:

Criterion of divisibility of the number three. What numbers does three divide into?

Tabla 10. Para identificar los números divisibles por el número tres

Múltiplos de tres						Observaciones por renglón
3	12	21	30	39	48	
6	15	24	33	42	51	
9	18	27	36	45	54	

Fuente: Elaboración propia

- ¿ How much are the digits in the first line?
- How much do the digits in the second line add up to?
- How much do the digits in the third row add up to?
- If the numbers increased, do you think the behavior would be the same?
- In general terms, mention what numbers the number three divides

Criterion of divisibility of the number five. What numbers does five divide into?

Tabla 11. Números del 1 al 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Fuente: Elaboración propia

Answer the following questions

- Taking into account the first 100 natural numbers, what numbers does five divide?
- What is the divisibility criterion for the number five?
- Do you know what the prime numbers are?
- Consult the biography of Eratosthenes at the following electronic address:
<https://es.wikipedia.org/w/index.php?title=Erat%C3%B3stenes&oldid=123632570>

There is no rule (formula) that allows to obtain them. According to Laracos (April 2, 2011), the procedure to be applied is described in table 9.

Tabla 12. Números del 1 al 100

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Fuente: Elaboración propia

Procedure to find the prime numbers less than 100

- a) The number one is not taken into account, this is called a unit number.
- b) Circle the number two and cross out all its multiples.
- c) Circle the number three and cross out all its multiples.
- d) Circle the number five and cross out all its multiples.
- e) Circle the number seven and cross out all its multiples.
- f) Circle the numbers that were not crossed out.
- g) Write all the numbers that were circled.

The fundamental theorem of arithmetic

It says that every nonzero natural integer can be decomposed as a product of prime factors in unique ways.

It should be noted that for the following activity the information from the video of Laracos Math (29 de enero de 2014).

Factor decomposition of composite numbers

Factor the number 30, using the prime numbers

Tabla 13. Factorización

30	2
15	3
5	5
1	

Fuente: Elaboración propia

So: $30 = 2 \times 3 \times 5$

Factor the following composite numbers

Tabla 14. Factorización de números compuestos

90 =	80 =	230 =
78 =	45 =	120 =
56 =	74 =	320 =

Fuente: Elaboración propia

Simplification of fractions

It means obtaining an irreducible fraction by dividing the numerator and denominator by the same number.

Simplify the following fractions

Tabla 15. Fracciones comunes

$\frac{150}{130} =$	$\frac{130}{140} =$	$\frac{200}{300} =$
$\frac{600}{580} =$	$\frac{656}{532} =$	$\frac{180}{200} =$
$\frac{1000}{800} =$	$\frac{900}{880} =$	$\frac{1500}{2800} =$
$\frac{2600}{3800} =$	$\frac{1500}{1280} =$	$\frac{1900}{1600} =$

Fuente: Elaboración propia

The least common multiple

The least common multiple (LCM) is the least common multiple of a certain number of numbers (see table 13).

Tabla 16. Números y múltiplos

Números	Múltiplos
4	4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80, 84...
6	6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96, 102 , 108, 114...

Fuente: Elaboración propia

- What are the common multiples of four and six?
- What is the least common multiple of them?

This number is known as the LCM. There are other procedures for calculating the LCM, such as prime factorization.

Calculate the least multiple of the following numbers

a) $\text{mcm}(12, 24, 14) =$

b) $\text{mcm}(3, 4, 5) =$

The numbers that are inside each parentheses are written to decompose them into factors, using only prime numbers. In this case, it matters too much that in each division the solver asks himself if any of the numbers he observes is divisible by the first prime number; if this is not possible, continue with the other prime numbers greater than two.

Tabla 17. Descomposición factorial del inciso *a)*

12, 24, 14	2
6, 12, 7	2
3, 6, 7	2
3, 3, 7	3
1, 1, 7	7
1, 1, 1	

Fuente: Elaboración propia

$$\text{mcm}(12, 24, 14) = 2 \times 2 \times 2 \times 3 \times 7 = 168$$

Tabla 18. Descomposición factorial del inciso *b*)

3, 4, 5	2
3, 2, 5	2
3, 1, 5	3
1, 1, 5	5
1, 1, 1	

Fuente: Elaboración propia

$$\text{mcm}(3, 4, 5) = 2 \times 2 \times 3 \times 5 = 60$$

Find the LCM of the following numbers (copy them in your notebook)

Tabla 19. Números compuestos

mcm (12, 48, 32) =	mcm (24, 12, 6) =	mcm (12, 24, 48) =	mcm (120, 6, 30) =
mcm (20, 10, 5) =	mcm (10, 30, 40) =	mcm (12, 48, 96) =	mcm (24, 12, 120) =

Fuente: Elaboración propia

Fractions that have different denominators

According to Carreon (July 24, 2016), the LCM is used, which is divided by the denominator of the first fraction and the quotient is multiplied by the numerator of the fraction. These operations are repeated with the other fractions, which will have the LCM as their denominator. The blue arrows indicate the operations that are performed with the first fraction.

$$\frac{3}{4} + \frac{1}{8} + \frac{7}{2} = \frac{6 + 1 + 28}{8} = \frac{35}{8} = 4\frac{3}{8}$$

Tabla 20. El cálculo del mcm

4, 8, 2	2	
2, 4, 1	2	$2 \times 2 \times 2 = 8$
1, 2, 1	2	
1, 1, 1		

Fuente: Elaboración propia

Look at the numerators obtained and the mcm (8).

$$\frac{6}{8}, \frac{1}{8} \text{ y } \frac{28}{8}$$

They are somewhat similar with the first fractions that were raised to add.

Solve the following sums in your notebook

Tabla 21. Suma de fracciones

a) $\frac{5}{4} + \frac{6}{6} + \frac{10}{2} =$	b) $\frac{10}{5} + \frac{4}{10} + \frac{8}{2} =$	c) $\frac{12}{6} + \frac{16}{8} + \frac{9}{10} =$	d) $\frac{4}{5} + \frac{9}{10} + \frac{6}{12} =$
e) $\frac{12}{5} + \frac{6}{4} + \frac{5}{20} =$	f) $\frac{9}{5} + \frac{7}{6} + \frac{6}{8} =$	g) $\frac{5}{30} + \frac{6}{60} + \frac{20}{12} =$	h) $\frac{7}{4} + \frac{9}{8} + \frac{14}{20} =$
i) $\frac{8}{3} + \frac{4}{6} + \frac{40}{9} =$	j) $\frac{8}{6} + \frac{7}{4} + \frac{16}{12} =$	k) $\frac{3}{4} + \frac{8}{12} + \frac{20}{24} =$	l) $\frac{5}{8} + \frac{6}{64} + \frac{4}{32} =$

Fuente: Elaboración propia

Mixed fractions

There are different methods. The one explained here is done in the following way: each mixed fraction is expressed to an improper one (multiplying the denominator by the whole part and adding the numerator, which is written as the numerator of the common fraction, whose denominator will be the same denominator that owns the fractional part).

Express a mixed fraction to an improper one

$$3\frac{1}{4} = \frac{13}{4}$$

$$3\frac{1}{4} - 2\frac{1}{3} - 4\frac{1}{2}$$

$$= \frac{13}{4} - \frac{7}{3} - \frac{9}{2} = \frac{39 - 28 - 54}{12} = \frac{39 - 82}{12} = \frac{-43}{12} = -3\frac{7}{12}$$

Do the following sums in your notebook

Tabla 22. Suma de fracciones mixtas

a) $6\frac{1}{2} + 7\frac{3}{3} + 8\frac{5}{4} =$	b) $4\frac{2}{3} + 6\frac{1}{6} + 8\frac{3}{9} =$	c) $2\frac{1}{6} + 3\frac{2}{9} =$	d) $6\frac{5}{7} + 8\frac{3}{7} + 12\frac{1}{7} =$
e) $6\frac{4}{9} + 4\frac{1}{3} + 5\frac{7}{6} =$	f) $5\frac{2}{3} - 4\frac{1}{4} =$	g) $\frac{7}{3} + \frac{3}{8} + \frac{12}{4} =$	h) $\frac{3}{5} + 7\frac{5}{8} + 3\frac{2}{3} =$

Fuente: Elaboración propia

Multiplications

The numerator is multiplied with numerator and denominator with denominator.

Some examples are seen in table 20.

Tabla 23. Ejemplos de multiplicación de fracciones

a) $\frac{5}{7} \times \frac{3}{2} = \frac{15}{14}$	b) $\frac{3}{8} \times 7 = \frac{3}{8} \times \frac{7}{1} = \frac{21}{8}$	c) $(-9)\frac{2}{3} = \left(\frac{-9}{1}\right)\frac{2}{3} = \frac{-18}{3} = -6$
---	---	--

Fuente: Elaboración propia

Solve the following multiplications

Tabla 24. Ejercicios de multiplicaciones de fracciones

a) $\frac{5}{9} \times \frac{18}{20} =$	b) $\frac{1}{5} \times \frac{5}{3} =$	c) $\frac{50}{25} \times \frac{100}{150} =$	d) $\frac{14}{18} \times \frac{9}{7} =$
e) $(12)\left(\frac{9}{7}\right) =$	f) $(15)\left(\frac{-4}{3}\right) =$	g) $(8)\left(\frac{-37}{3}\right) =$	h) $\left(\frac{13}{3}\right)(8) =$

Fuente: Elaboración propia

Divisions

These are multiplied in a cross way. Some examples are shown in table 22.

Tabla 25. Ejemplos de divisiones aritméticas

a) $\frac{3}{6} \div \frac{2}{5} = \frac{15}{12} = \frac{5}{4}$	b) $6 \div \frac{2}{3} = \frac{6}{1} \div \frac{2}{3} = \frac{18}{2} = 9$	c) $4 \div \frac{2}{9} = \frac{4}{1} \div \frac{2}{9} = \frac{36}{2} = 18$
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Fuente: Elaboración propia

Solve the following divisions

Tabla 26. Ejercicios de divisiones aritméticas

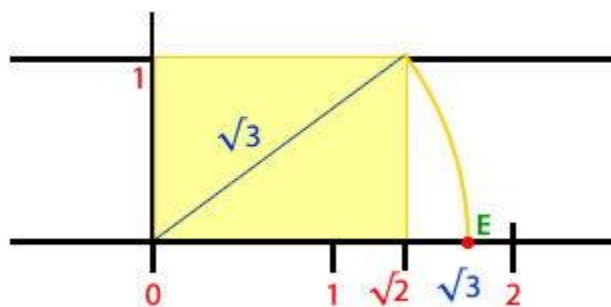
a) $\frac{24}{10} \div \frac{15}{30} =$	b) $\frac{5}{3} \div \frac{9}{20} =$	c) $\frac{18}{36} \div \frac{1}{3} =$
d) $\frac{11}{34} \div \frac{8}{6} =$	e) $\frac{13}{4} \div \frac{7}{6} =$	f) $\frac{12}{15} \div \frac{-3}{-4} =$

Fuente: Elaboración propia

Irrational

At this point, what Scaglia (2000) mentions about cognitive conflict is resumed for the treatment of rational numbers on the number line.

Figura 13. Números irracionales



Fuente: <https://www.portaleducativo.net/segundo-medio/4/ordenar-numeros-irracionales-representarlos-en-recta-numerica>

Until the creation of the rational numbers, it seemed that you already had all the numbers and none were missing. According to Valdez (2008), in the time of Pythagoras of Samos, a Greek mathematician who lived approximately in the years 569 and 475 BC. C., the Pythagorean school argued that the numbering reached that point, however, history comments that given a calculation problem a result did not belong to the known numbers.

The problem was the following: "Given an isosceles right triangle of magnitude equal to unity, calculate the measure of its hypotenuse."

Tabla 27. El triángulo rectángulo y el teorema de Pitágoras

Datos	Fórmula	Sustitución	Análisis
El cateto a = 1	$c^2 = a^2 + b^2$	$c = \sqrt{1^2 + 1^2}$	El valor de la $\sqrt{2}$ no se puede expresar como una fracción.
El cateto b = 1	Despejar a c	$c = \sqrt{1 + 1}$	
El cateto c =	$c = \sqrt{a^2 + b^2}$	$c = \sqrt{2}$	

Fuente: elaboración propia

Reflection: So, looking for the square roots of other numbers, we realize that something similar happens with the $\sqrt{2}$, another of the rational numbers is the value of π , which also has non-periodic decimals and the Euler value, with an approximate value of 2.7181.

Operations will not be carried out in this numerical field, since it corresponds to the upper secondary level and, in addition, the time allocated to solve this sequence corresponds to a time outside of school hours.

Results of the teaching intervention

To analyze the results, two study plans that are linked to teacher training were taken into account (Ministry of Public Education [SEP], 2011, 2017a). The didactic sequence proposed to the students was satisfactory due to the participation of all the students, who were answering each of the questions and solving the operations.

Due to the structure of the didactic sequence, the role of the teacher was always that of facilitator, while the students had an active and constructivist participation.

It is important to mention that the work of Baldor (1996) was important to contextualize the work of this article. The same was the document Model educational for compulsory education. Educate for freedom and creativity (SEP, 2017b).

Discussion

Before the application of the didactic sequence, there was a wide expectation of curiosity on the part of the students, mainly due to the possible difficulty of the subject; However, when they received the worksheets and read the activities, they realized that they not only produced new knowledge, but also reinforced what they already had, that is, with this sequence they systematized their information, but, in addition, they learned in a way general why other numeric fields were created.

The limitation found in the present work is the scarce reproducibility that can be done at school to determine some other factors that affect learning, which would provide more information for the article. If it were a high school, with six groups of the same grade, surely more information would be integrated into the work that is presented.

The number field where the students started to have trouble was in whole numbers. Here it was specified that preferably, as future teachers, they should not go to the next field because, if they neglected it, they would have many difficulties in solving the linear and quadratic equations, since the error of signs would cause that the problems raised could not be solved.

As for rational numbers, it has always been a headache to learn them, but now, despite some difficulties, they were better supported with prior knowledge of divisibility criteria, prime numbers, and the factoring of composite numbers.

As a strength of the research work, the wide disposition of the students to carry out the proposed activities was observed, as well as the systematization of the work so that the first ones were support of the following ones, this meant that, at the end of the academic work, their numerical ideas were better organized.

Regarding the area of weaknesses, it can be mentioned that future teachers of telesecundaria schools on several occasions approach content of the level in a superficial way, without detracting from the educational quality. That is why teacher educators have to do substantial work in small spaces of time, in order to be productive.

Conclusions

The work carried out with the students of the Bachelor of Telesecundaria Telesecundaria was beneficial for their professional training, since, from now on, in each mathematical intervention, they will know what numerical field they are talking about and what rules must be applied properly. This is extremely important for troubleshooting.

Teachers who do not know the classification of real numbers tend to make mistakes in mathematical operations, so it is essential that they know the rules that are established in each numerical field.

The students were able to differentiate the various numerical fields. Now you know how the real numbers are organized. That in each field there is a rule that governs the operations that are carried out there, for example, $7-2$ is not the same as $2-7$; For the first case, the result is $+5$ and for the second it is -5 , these results represent different things, namely, in the first case, if we are talking about monetary units, it means that a certain person owns five pesos, while in the first case, second case, the meaning is that the person has a debt of five pesos.

Now the normalist student has more records to perform operations, not only the numerical and verbal, but also the graph, which in this case is represented on a number line, which especially supports the addition and subtraction of signed numbers.

It establishes a classification of the fractions according to the denominator that it has; You know that if the denominator is less than the unit, it is called a proper fraction, and if it is equal to or greater than the unit it is known as an improper fraction.

The construction of the divisibility criteria was given with the following numbers: two, three and five, making operations, observations and conjectures about the division of the aforementioned numbers.

With the previous actions, the prime numbers (those with only two divisors) less than 100 were identified and they were notified that, to date, there is no formula to help calculate them; there is a procedure that was designed by Eratosthenes, who was a Greek mathematician who died in 194 BC. C. and that one of his most recognized calculations is having mentioned the length of the circumference of the Earth.

With the use of prime numbers, compounds (those that have more than two divisors) can be decomposed into prime factors; This action facilitates the operations of addition and subtraction fractions.



As can be seen, the didactic sequence consisted of chaining situations that allowed to lay the foundations of arithmetic for the students of the Prof. Rafael Ramírez Federal Urban Normal School.

References

- Baldor, A. (1996). *Aritmética*. México: Publicaciones Cultural.
- Carreon, D. (24 de julio de 2016). Suma de fracciones con diferente denominador. (video de YouTube). <https://www.youtube.com/watch?v=LVHo5xvsvO0>. Recuperado de <https://www.youtube.com/watch?v=LVHo5xvsvO0>.
- Céspedes, E. (27 de septiembre de 2017). Balanza comercial. ABCfinanzas. Recuperado de <https://www.abcfinanze.com/principios-de-economia/que-es-la-balanza-comercial>.
- Laracos Math. (2 de abril de 2011). Números primos con la criba de Eratóstenes. (video de YouTube). Recuperado de <https://www.youtube.com/watch?v=XytXXaKytVU>.
- Laracos Math. (29 de enero de 2014). Factorización de números a factores primos. (video de YouTube). Recuperado de <https://www.youtube.com/watch?v=Yul9zO2ONdQ>.
- MateFacil. (8 de agosto de 2016). Clasificación de números: naturales, enteros, racionales, irracionales, reales, complejos. (video de YouTube). Recuperado de https://www.youtube.com/watch?v=rtNC7g1h_JA.
- Matemáticas profe Alex. (28 de octubre de 2018). Comprendiendo la suma y resta de fracciones | Explicación completa. (video de YouTube). Recuperado de <https://www.youtube.com/watch?v=YpSb9LlsFv8>.
- Méndez, Z. (1993). *Aprendizaje y cognición*. Barcelona, España: Universidad Estatal a Distancia.
- Mora, L. (2004). *Concepciones de estudiantes de licenciatura en matemáticas sobre números reales*. (tesis de maestría). Universidad Pedagógica Nacional, Bogotá.
- Rodríguez, E. (s. f.). Juegos con fracciones: lectura, comparación, fracciones. (juego educativo). Recuperado de https://www.tes.com/lessons/zABJh2K_GmWTdg/fracciones.
- Sáenz, E. (22 de abril de 2019). ¿Qué son realmente los números reales? (video de YouTube). Universitat Politècnica de València, España. Recuperado de <https://www.youtube.com/watch?v=xOjQ3u7jSLQ>.

- Sánchez, D. (2 de febrero de 2014). Las matemáticas en la prehistoria 5ª parte: Los huesos de Lebombo y de Ishango. *Prehistoria al Día*. Recuperado de <https://prehistorialdia.blogspot.com/2014/02/las-matematicas-en-la-prehistoria-5.html>.
- Scaglia, S. (2000). *Dos conflictos al representar números reales*. (tesis doctoral). Universidad de Granada, España.
- Secretaría de Educación Pública [SEP]. (2011). *Programas de Estudio 2011. Guía para el maestro. Educación básica secundaria*. México: CONALITEG.
- Secretaría de Educación Pública [SEP]. (2017a). *Aprendizajes clave para la educación integral. Plan y programas de estudio para la educación básica*. México: Secretaría de Educación Pública.
- Secretaría de Educación Pública [SEP]. (2017b). *Modelo educativo para la educación obligatoria. Educar para la libertad y la creatividad*. México: Secretaría de Educación Pública.
- Struik, D. (1998). *Historia concisa de las matemáticas*. México: Instituto Politécnico Nacional.
- Tu profe en línea. (13 de mayo de 2015). Clasificación de Fracciones – Completo. (video de YouTube). Recuperado de <https://www.youtube.com/watch?v=2qqoiCXF6YI>.
- Valdez, V. (2008). *Los conjuntos numéricos a través de la historia*. (tesina). Instituto Superior de Fundación Suzuki, Argentina.
- Wikipedia. (30 de abril de 2018). Eratóstenes. Recuperado de <https://es.wikipedia.org/w/index.php?title=Erat%C3%B3stenes&oldid=123632570>.
- Wikipedia. (22 de abril de 2019). Número real. Recuperado de https://es.wikipedia.org/w/index.php?title=N%C3%BAmero_real&oldid=123835447.
- Wikipedia. (26 de noviembre de 2019). Papio. Recuperado de <https://es.wikipedia.org/wiki/Papio#/media/File:Olivebaboon.jpg>.