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Perforación direccional, geometría analítica, trigonometría y GeoGebra

Directional drilling, analytical geometry, trigonometry and GeoGebra

Perfuração direcional, geometria analítica, trigonometria e GeoGebra

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Resumen

En un mundo globalizado y moderno como el de hoy en día, el consumo del petróleo y gas, forma parte esencial de actividades sociales, económicas e industriales, ha obligado a las empresas petroleras a realizar exploraciones en lugares más inhóspitos y de difícil acceso, exponiendo herramientas, equipos y personal a condiciones extremas tanto en operaciones y condiciones climáticas, significando un mayor riesgo y exigiendo a los ingenieros de diseño de pozos a desarrollar nuevos métodos de perforación, como por ejemplo: la perforación vertical, direccional, horizontal y multilaterales. En México, la región petrolera terrestre la integran los estados de Tabasco, Campeche, Veracruz y Tamaulipas, la región marina se extiende sobre el Golfo de México.

La perforación direccional es uno de los métodos más utilizados en la exploración de los hidrocarburos, académicamente muy pocas instituciones de educación superior que ofertan la carrera de Ingeniería Petrolera poseen equipos, simuladores y software especializado que permitan asegurar un proceso adecuado de enseñanza y aprendizaje. En el presente artículo se muestra la aplicación de las matemáticas básicas como el teorema de Pitágoras, la trigonometría y la geometría, aplicada en la descripción y esquematización de la trayectoria de una perforación direccional, acompañado de un recurso desarrollado en GeoGebra, que se caracteriza por ser un programa de matemáticas libre, que puede utilizarse en dispositivos móviles, incluso sin conexión a Internet, está disponible en diversas plataformas, permite trabajar en 2D y 3D. En él, se logró integrar los conceptos básicos de la perforación direccional y las variables que permiten esquematizar su trayectoria.

Palabras claves: Desviación, kop, azimut, direccional, GeoGebra.

Abstract

In a globalized and modern world like today, the consumption of oil and gas, which is an essential part of social, economic, and industrial activities, has forced oil companies to explore more inhospitable and difficult-to-access places, exposing tools, equipment, and personnel to extreme conditions in both operations and weather conditions, meaning greater risk and requiring oil well design engineers to develop new drilling methods, such as; vertical, directional, horizontal and multilateral drilling. In Mexico, the terrestrial oil region comprises the states of Tabasco, Campeche, Veracruz, and Tamaulipas. The marine region extends over the Gulf of Mexico.

Directional drilling is one of the most used methods in exploring oil wells. Academically, very few higher education institutions that offer a career in petroleum engineering have specialized equipment, simulators, and software that allow them to ensure their teaching and learning. This article shows the application of basic mathematics, such as the Pythagorean theorem, trigonometry, and geometry, applied in the description and schematization of the trajectory of directional drilling, accompanied by a resource developed in GeoGebra, which is used because it is a free mathematics application that can be used on mobile devices, even without an internet connection, it is available on various platforms. It allows you to work in 2D and 3D. It was possible to integrate the basic concepts of directional drilling and the variables enabling its trajectory to be schematized.

Keywords: Deviation, kop, azimuth, directional, GeoGebra.

Resumo

Em um mundo globalizado e moderno como o de hoje, o consumo de petróleo e gás, que é parte essencial das atividades sociais, econômicas e industriais, forçou as empresas petrolíferas a explorar lugares mais inóspitos e de difícil acesso, expondo ferramentas, equipamentos e pessoal a condições extremas, tanto nas operações quanto nas condições climáticas, o que significa um risco maior e exige que os engenheiros de projeto de poços de petróleo desenvolvam novos métodos de perfuração, como a perfuração vertical, direcional, horizontal e multilateral. No México, a região petrolífera terrestre compreende os estados de Tabasco, Campeche, Veracruz e Tamaulipas. A região marinha se estende pelo Golfo do México.

A perfuração direcional é um dos métodos mais usados na exploração de poços de petróleo. Do ponto de vista acadêmico, pouquíssimas instituições de ensino superior que oferecem uma carreira em engenharia de petróleo têm equipamentos especializados, simuladores e softwares que lhes permitem garantir o ensino e o aprendizado. Este artigo mostra a aplicação da matemática básica, como o teorema de Pitágoras, a trigonometria e a geometria, aplicada na descrição e esquematização da trajetória de perfuração direcional, acompanhada de um recurso desenvolvido no GeoGebra, que é utilizado por ser um aplicativo matemático gratuito que pode ser usado em dispositivos móveis, mesmo sem conexão com a internet, estando disponível em várias plataformas. Ele permite que você trabalhe em 2D e 3D. Foi possível integrar os conceitos básicos de perfuração direcional e as variáveis que permitem esquematizar sua trajetória.

Palavras-chave: Desvio, kop, azimute, direcional, GeoGebra.

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Introduction

Hydrocarbon exploitation is important in the economy of any oil-producing country, and Mexico is no exception (Damian, 2015). The construction of oil wells is a multidisciplinary process since it requires diverse knowledge, such as mathematics, physics, chemistry, geology, hydraulics, materials science, and experience in the work field.

An oil well is the only means a reservoir can communicate with the surface. Its drilling and casing are generally carried out with drill bits and casing pipes of different diameters and depths. Oil well drilling began with the percussion method. This method was slow since some formations had a higher degree of consolidation, causing this process to slow down, which is why it was used for shallow wells. In 1901, rotary drilling began, where the drill bit was responsible for cutting, fracturing, and crushing the different formations. It also used a medium known as drilling fluid to transport the solids generated to the surface. Between 1983 and 1984, the first direct current Top Drive appeared. In 1987, the hydraulic model was introduced, providing excellent pipe handling safety, among other advantages.

The increased demand for hydrocarbons led to increased production volumes and new explorations in inaccessible areas, drilling wells along coastlines, and establishing multiple wells from offshore or onshore platforms, among other actions. This resulted in the adoption of directional drilling. Initially, this technique was used as a corrective operation to free stuck tools, correct accidental deviations of wells, or drill nearby relief wells to control blowouts (Petróleos de Venezuela SA, PDVSA, 1996). It originated in the 1920s in the USA, but it was not until 1929 that precise measurement systems were used in the Seminole fields in Oklahoma. However, the first well drilled in a directionally controlled manner was in 1930 in Huntington Beach, California (Damian, 2015). In Mexico, there are no records of controlled directional drilling; the first case corresponds to 1960 in Choapas, Veracruz (Hernández, 2018). However, Muñoz (2014) describes that in Mexico, directional wells have been successfully drilled in the Cuitláhuac, Agua Fría, Cerro Nanchital, Catedral, Chincotepec, Burgos fields, to mention a few.

At the national level, there are about 58 higher education institutions that offer a career in petroleum engineering; only in Tabasco are there approximately 28, of which only a reduced percentage have simulators for oil well drilling and control, but all of them lack software for design, planning, control, and analysis of anti-collision, directional drilling, such as DSP ONE. Only a few oil companies have it, given their oil well drilling activities and licensing costs. This work aims to show the first application of GeoGebra with the integration of analytical geometry, trigonometry, and the theory of directional drilling.

It is essential to know the schematization of the well in each stage, whether vertical or directional; it is called mechanical state. In this state, the different diameters and settlement depths of the casing pipes are identified, as well as the tools that make up the drilling string; in the directional case, its degree of difficulty corresponds to the schematization of the sections: vertical, construction angle and target, the various numerical calculations to determine the values of radius, maximum angle, length of the construction arc, length to the target, among others, therefore, GeoGebra allows to schematize 2D and 3D graphics, as well as the linking of the variables between the main sections, which reinforces the comments of Trigueros and Martínez-Planell (2010), the understanding of the geometric and algebraic properties of the functions are related to the visualization.

Development

The main objective of oil well drilling is to make a hole that starts on the surface and ends in the target or producing zone; regardless of its depth, the hole must have (be the means with) the capacity to transport the hydrocarbon to the surface, its cycle has the following stages: drilling, completion, information collection, maintenance and plugging, the geological characteristics of each of the formations of interest and those of the hydrocarbon allow to establish if this will be vertical or directional, the first is the most common, its development represents greater profitability, both in the elimination of cuttings, as in pipe and cement; while the directional, consists of maintaining control of the direction and inclination of the well towards the previously determined underground geological objective. Directional drilling has greater scope in the formations or producing zones, reduces the number of wells, improves the project's profitability, and reduces the environmental impact (Thorhallsson, 2012). There are 18 or more calculation techniques to determine the hole trajectory, for example, average angle, radius of curvature, minimum curvature, tangential, balanced tangential, and others; among their differences are the straight-line approximations and curve segments (Schlumberger, 2010; Girling, 2008; Barros, 2008). Directional trajectories are classified into types J, S, and modified S, as shown in Figures 1 and 2.

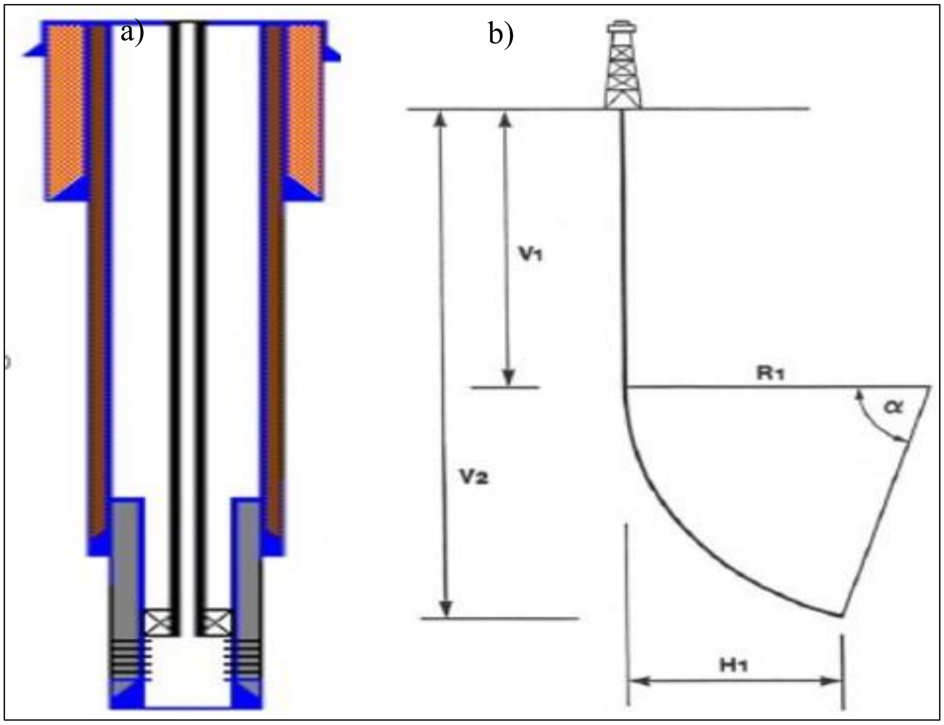

Figure 1. a) Vertical well; b) Directional wells Types: J

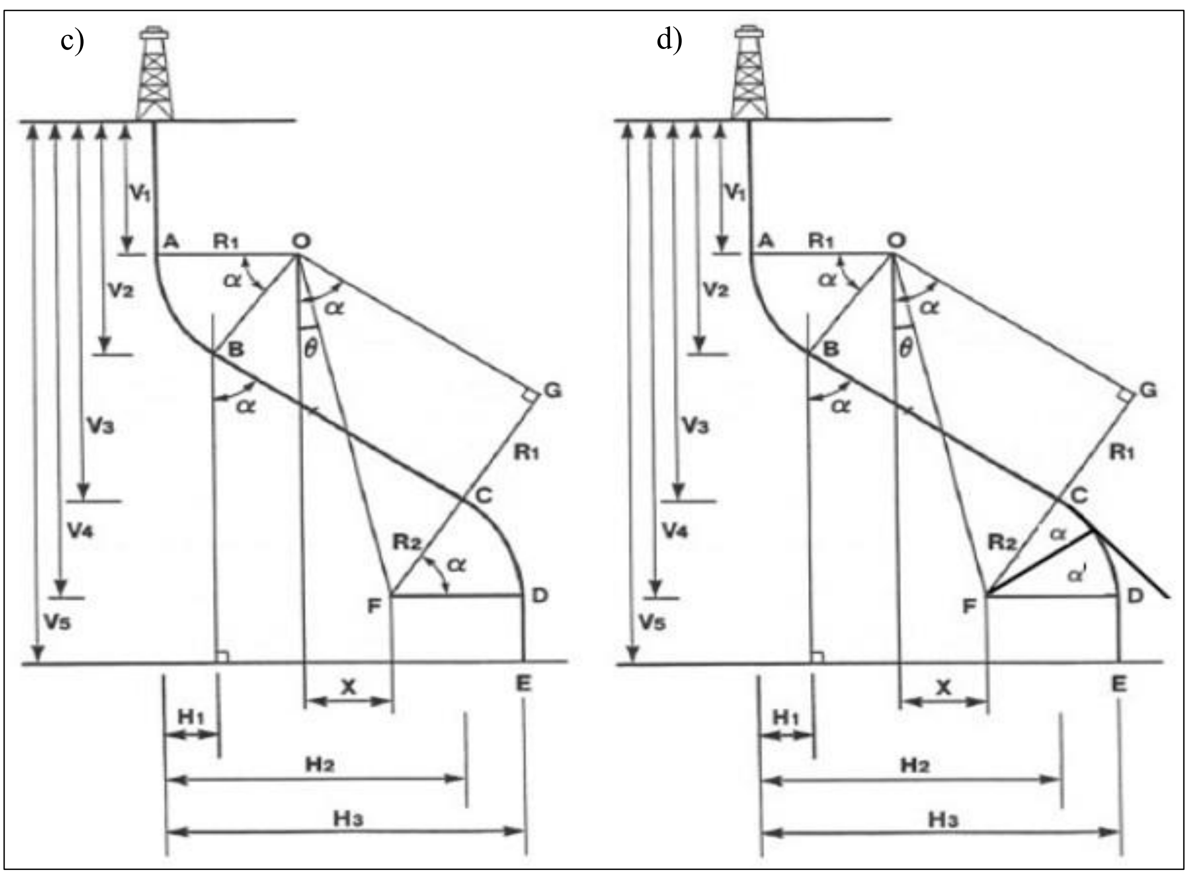

Figure 2. c) S-type directional well and d) Modified S (Salazar, 2011).

Salazar (2011) considers that the radius of curvature in directional drilling is classified according to its length: long (angle of 2° to 6°), medium (angle of 8° to 50°), and short (greater than 50°) for every 30 meters drilled, as shown in Figure 3.

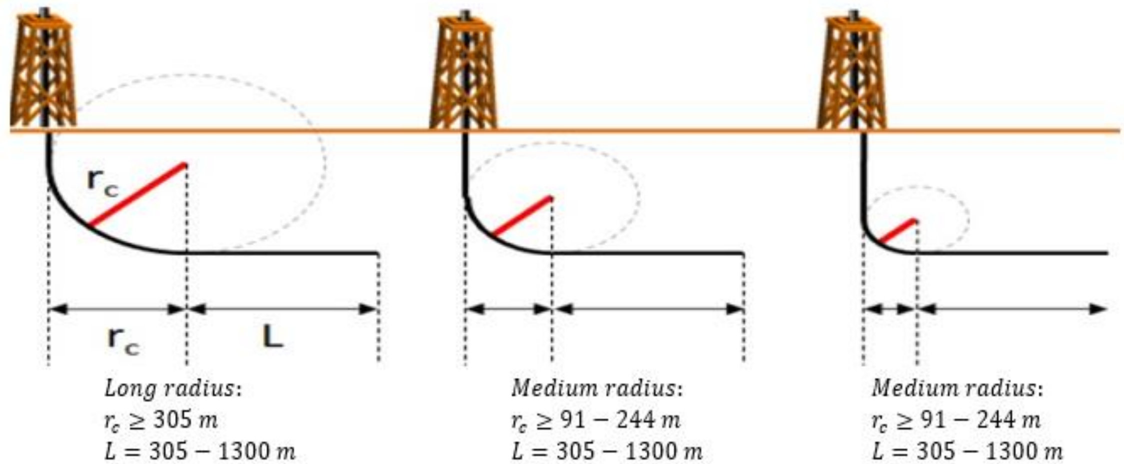

Figure 3. Construction radii as a function of their length (Salazar, 2011).

Elements of directional drilling,

According to Flores (2011) and Cozar (2017), the elements of directional drilling are:

Kickoff point (KOP), or starting point, is where the vertical section begins its new horizontal section.

The End of Buildup (EOB), or end of increment, is where the buildup section ends, the wellbore portion where the decline angle gradually increases.

EOD (End of Drop -end of fall) is where the drop or decay section ends.

Azimuth, or bearing, is the angle in degrees measured from the horizontal component in the plane of a wellbore.

Dog Leg is a severe adjustment or change in the angle of the hole direction.

Offset is the horizontal distance between vertical lines passing through the target in the subsurface and the wellhead.

The True Vertical Depth (TVD) is the vertical depth measured from a surface reference point to a survey point along the wellbore trajectory.

The Developed Depth (MD) is the depth measured from the surface to a point on the wellbore trajectory.

Analytical geometry and trigonometry

Trigonometry refers to the measurement of triangles, which in turn relate to angles, right triangles, trigonometric functions, and arc length, among other things.

An angle is formed by two lines or rays that have a common end called a vertex. The measurement of an angle in degrees is based on the assignment of 360°. It is sometimes useful to convert decimal numbers to degrees, minutes, seconds, and vice versa.

In geometry, angles are classified in various ways. For instance, an acute angle varies between 0º and 90º, a right angle ranges between 90º, and an obtuse angle varies between 90º and 180º. Two acute angles are said to be complementary if their sum is 90º, and two positive angles are supplementary if their sum is 180º.

The length or distance is a fundamental part of well drilling since it determines the number of meters of pipe to be used. However, the pipe does not always run in a straight line, so it is necessary to know how to calculate the arc length, which is as follows: $s = r\Theta$, where s is the arc length, r is the radius, and Θ is the angle in radians.

GeoGebra in education

Among the various current programs used in engineering education are Mathlab, Mathematica, and Cabri Geometry, which usually require paid licenses; sometimes, their use and functions are restricted to computer laboratory environments (Coca and Benítez, 2023). GeoGebra integrates different areas such as, for example, algebra, geometry, trigonometry, analytical geometry, differential, integral, and statistics calculus, as well as the efficient visualization and manipulation of geometric objects (Alkhateeb and Al-Duwairi, 2019; Emaikwu et al., 2015; Soetadianta, 2014). Additionally, GeoGebra offers a version for mobile devices that can be used in the classroom, even without an Internet connection (Fioriti, 2017; Ibarra, 2019); it is available for various platforms: macOS, Windows, Linux, Android, and iOS (Ramírez, 2020).

Carrillo and Llamas (2009) considered that the primary use of GeoGebra focuses on Euclidean and analytical geometry.

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Results

Graphical representation of directional drilling

a) Initially, the Cartesian plane is drawn in the first quadrant, in which the "Y" axis represents the depth of the well and the "X" axis represents the displacement that it will have on the horizontal, which indicates that it is directional drilling.

b) The length of the vertical perforation, which will be called point "A," is located on the "Y" axis.

c) The point representing the target (objective, i.e., the final point of the trajectory) will be located. This point will be the intersection of the target distance ("X" axis) with the target TVD ("Y" axis) and will be represented with "C," as shown in Figure 4.

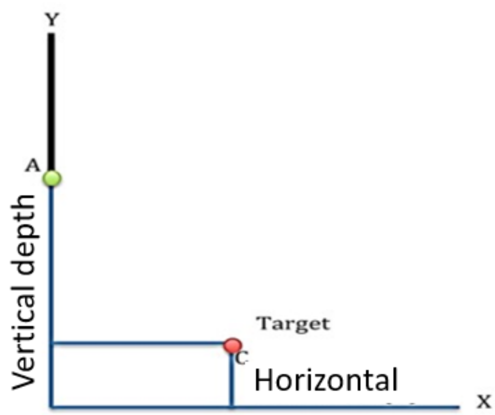

Figure 4. Vertical section and target of directional drilling. Source: Own elaboration

d) The value of R_C (construction radius) is calculated and located. The value will be the center of the semicircle and will be named “0”**.**

e) With $R_{C,}$ a semicircle is delineated and whose diameter will be $2R_C$. This will start from the KOP value, as shown in Figure 5.

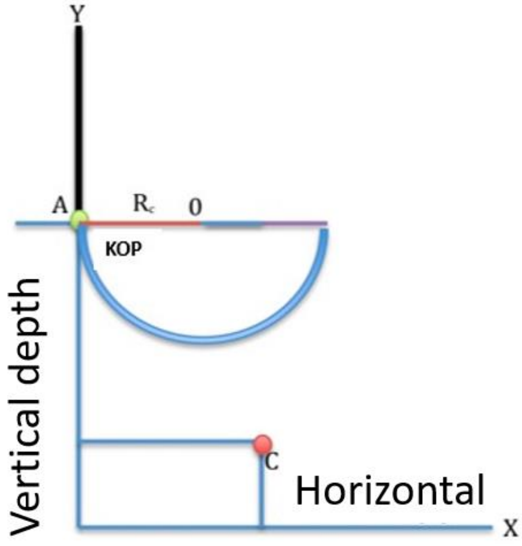

Figure 5. Location of the construction radius, origin and KOP. Source: Own elaboration

f) A line is drawn from point C (target), which is tangent to the semicircle, and the point it intercepts will be called "B"**.**

g) At point "B" a straight line will be drawn parallel to the "Y" axis and will be represented by "EOB", This straight line will cut the line R_C, whose intersection will be denoted as "F".

h) Starting from point "C", a straight line will be drawn parallel to the "Y" axis, resulting in a second right triangle, with segments: 0C, 0D, CD, as seen in Figure 6.

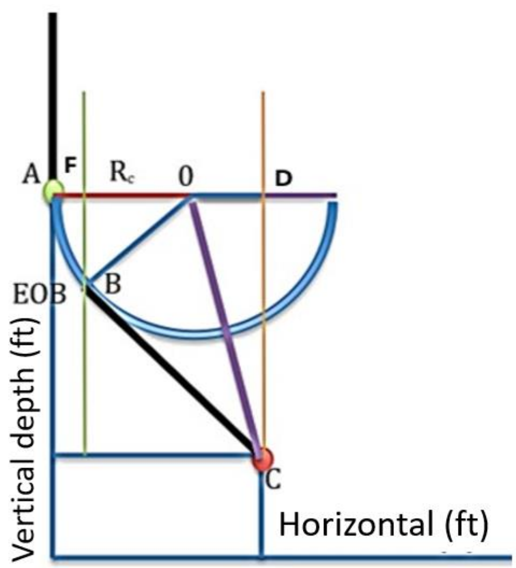

Figure 6. Directional path end point, KOP and construction radius. Source: Own elaboration

i) The points must be joined, forming the segments 0A= R_C, 0B, 0C, and CB, giving a right triangle and an arc.

j) The interior angles and the value of their axes must be calculated for each right triangle, as shown in Figure 7.

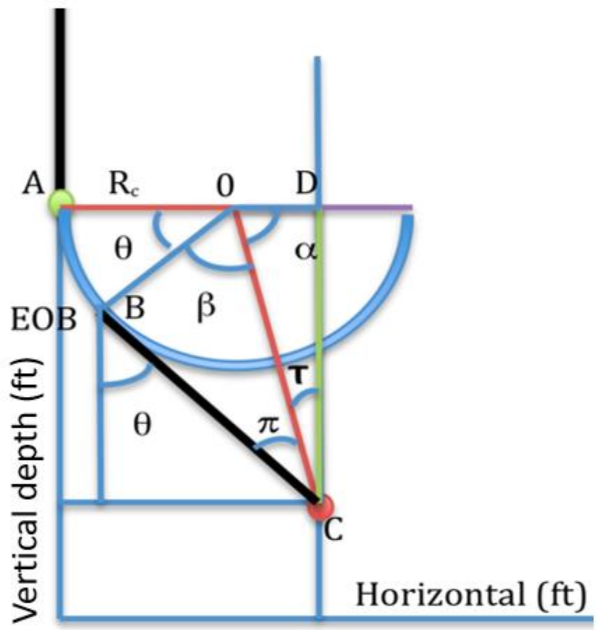

Figure 7. Directional drilling element, KOP, PVV, KOP, construction radius. Source: Own elaboration

Figure 8 shows the numerical calculations for a directional well, "Type J" where the following data: Maximum inclination angle = 41.83°; KOP = 6925 ft; BUR = 3°/100 ft; Target TVD = 10500 ft; Target distance = 2500 ft; Target direction = S 28° E, will allow finding the following:

- TVD at the end of the construction phase:
- Depth Measured at the end of the construction phase:
- Depth Measured to Target

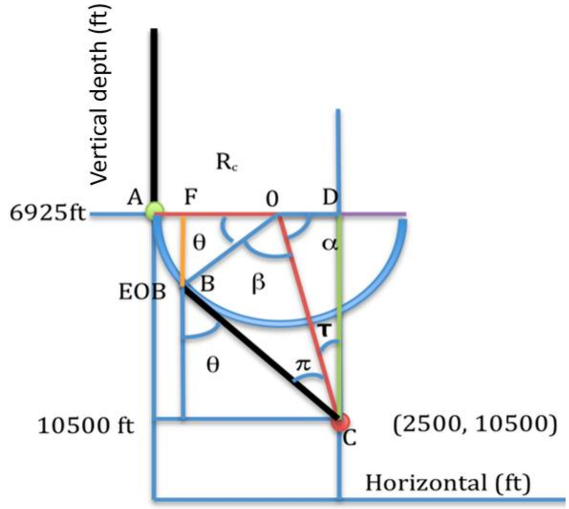

Figure 8. Directional drilling exercise, Source: Own elaboration

The first step is to find the value of the radius of the semicircle R_C using equation (1):

$$BUR = \frac{5729.6}{R_c} \quad (1)$$

Clearing R_C from equation (1) and replacing BUR = 3°/100 ft, results in:

R_C =1909.87 ft

From Figure 7, segment AD measures 2500 ft so segment 0D measures:

$$\overline{0D} = \overline{AD} - R_c = 2500 - 1909.87 = 590.13\, ft \quad (2)$$

From the right triangle with vertices $0, D$ and C:

$$\overline{DC} = 10500 - 6925 = 3575$$

and by applying the Pythagorean theorem it turns out that:

$$\overline{0C} = \sqrt{(0D)^2 + (DC)^2} = \sqrt{(590.13\, ft)^2 + (3575\, ft)^2} = 3623.37\, ft \quad (3)$$

On the other hand, the cosine of the angle α of the triangle $0CD$ is given by

$$cos\alpha = \frac{\overline{0D}}{\overline{0C}} = \frac{590.13\, ft}{3623.37\, ft} = 0.16 \quad (4)$$

Taking the inverse cosine of this number gives:

$$\alpha = cos^{-1}0.16 = 80.62 \cong 80^{\circ}\, 37'\, 35.8''.$$

The sum of the interior angles of a triangle is 180º, so the ∢τ of $\Delta 0DC$ is:

$$\tau = 180^{\circ} - 90^{\circ} - 80° \, 37' \, 35.8'' = 9° \, 12' \, 24.83. \quad (5)$$

Now, considering the $\Delta 0BC$. Note that:

$$\overline{0B} = R_c = 1909.86 \, ft \text{and } \overline{0C} = 3623.37 \text{ ft}$$

As Δ0BC It is a right triangle (because every straight-line tangent to the circle is perpendicular to its radius), then by the Pythagorean theorem the segment BC is given by:

$$\overline{BC} = \sqrt{(0C)^2 + (0B)^2} = \sqrt{(3623.37 \, ft)^2 - (1909.87 \, ft)^2} = 3079.15 \, ft. \quad (6)$$

Note that segment BC is the opposite leg of angle β, consequently:

$$sen \, \beta = \frac{\overline{BC}}{\overline{0C}} = \frac{3079.15 \, ft}{3623.37 \, ft} = 0.84 \Rightarrow \beta = sen^{-1}(0.84) = 58.19 \cong 58^{\circ} \, 11' \, 26.82''$$

The ∢0BC measures 90º so we can find the angle π as follows:

$$180^{\circ} = 90^{\circ} + 58^{\circ} \, 11' \, 26.13'' + \pi \Rightarrow \pi = 31.80 \cong 31^{\circ} \, 48' \, 35.18'' \quad (7)$$

Looking at Figure 1 and remembering that every semicircle has a $\sphericalangle = 180^{\circ}$:

$$\theta + \beta + \alpha = 180^{\circ} (8)$$

Hence:

$$\theta = 180^{\circ} - \beta - \alpha = 180^{\circ} - 58^{\circ} \, 11' \, 26.13'' - 80^{\circ} \, 37' \, 35.8'' = 41.18 \cong 41^{\circ} \, 10' \, 58.07'',$$

the latter being the maximum angle of inclination.

Now, the depth at the end of the construction stage, that is, the length of the arc AB. The angle must be converted from degrees to radians θ, i.e.:

$$\theta = \frac{41 \times \pi}{180} = 0.71 \text{ radians}$$

Then the arc length AB is given by:

$$s = R_c \times \theta = 1909.87 \times 0.71 = 1372.77\,ft \quad (9)$$

Consequently, the depth measured at the end of the construction stage is given by:

$$Vertical\ depth + arc\ AB = 6925 + 1372.77 = 8297.77\,ft$$

So, the total depth measured to the target is:

$$Vertical\ depth + arc\ AB + \overline{BC} = 6925 + 1372.77 + 3079.15$$
$$= 11376.92\,ft$$

The length of segment FB which is the opposite leg to the angle θat ΔF0B:

$$sen\ \theta = \frac{\overline{FB}}{\overline{0B}} \quad (10)$$

Hence:

$$\overline{FB} = (sen\ 41^{\circ})(1909.87\,ft) = 1252.98\,ft$$

$$TVD\ in\ the\ end\ of\ construction\ stage = 6925\,ft + 1252.98\,ft = 8177.98\,ft$$

GeoGebra application development

Points A (0,0) and B (0,-6925) form the vertical section or vertical drilling, with a depth of 6925 ft, point C (1909.87, -6925) is the center of the circumference, and segment CB corresponds to the construction radius with a value of 1909.87 ft, point D (2500,-10500) is the target point, this point joins point B´, the angle formed by points BCB´ forms the angle of 41.83° called the construction angle, these are the input data, Figure 9 schematizes the initial points of the directional trajectory.

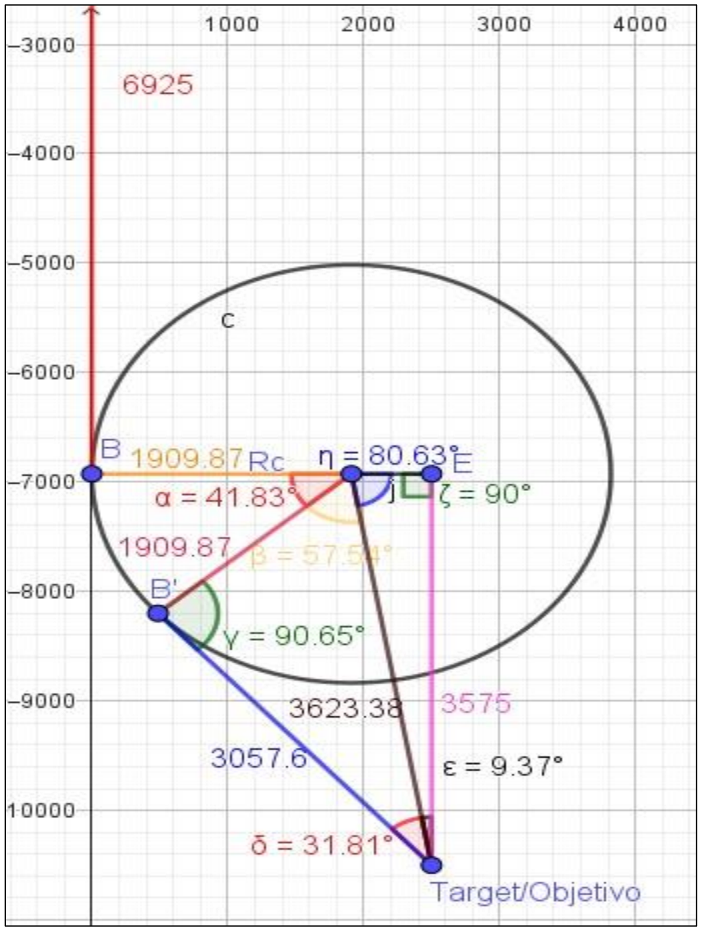

Figure 9. Initial schematization of the trajectory with GeoGebra.

Source: Own elaboration

In Figure 10, the interior angles of triangles B´CD and CED are shown with their respective values for each side that comprise it. In Table 1, the angles are grouped according to the corresponding triangle.

Table 1. Interior angles of ΔB´CD and ΔCED

α=41.83° formed by the points: BCB´	angle, construction angle
β=57.54° formed by the points: B´CD	The interior angles β, μ and δ correspond to ΔB´CD. Whose sides $\overline{BC}, \overline{CD}\ y\ \overline{B'D}$.
μ=90.65° formed by the points: DB´C	
δ=31.81° formed by the points: CDB´	
€=9.37° formed by the points: EDC	The interior angles €, ζ and η correspond to ΔCED. Whose sides $\overline{CE}, \overline{ED}\ y\ \overline{CD}$.
ζ=90° formed by the points: CED	
η=80.63° formed by the points: DCE	

Source: Own elaboration

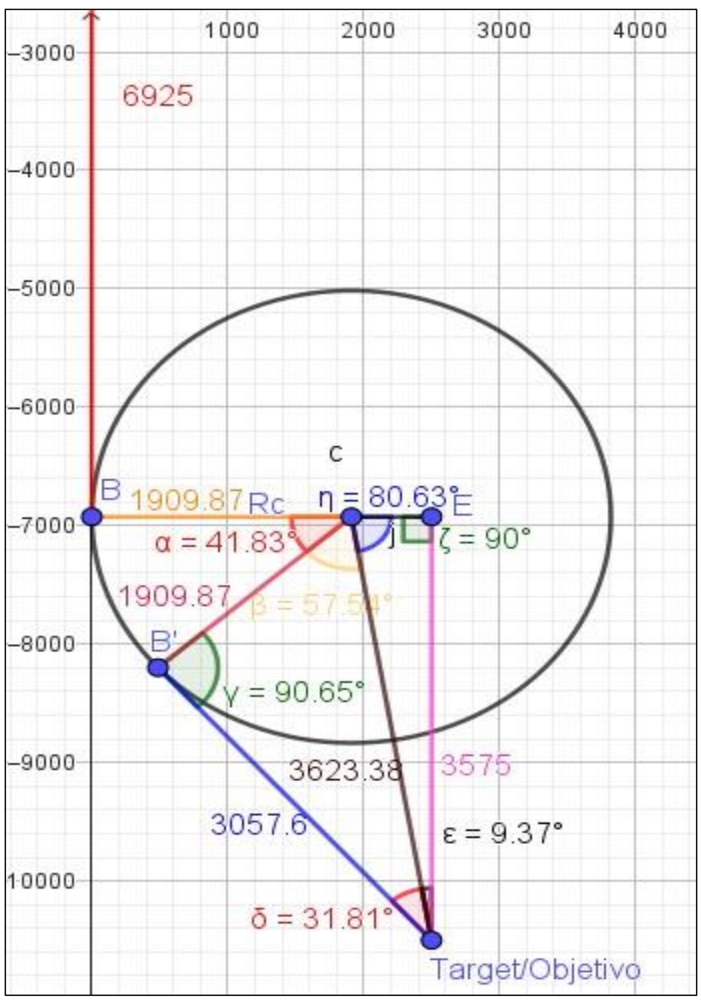

Figure 10. Magnitude of each of the components of the vertical sections, construction angle and target.

Source: Own elaboration

In the following link, the complete exercise developed in GeoGebra is presented. The animation will be seen by simply moving the slider button: geogebra.org/calculator/xjgys3xn.

Discussion

Among the previous works, we can mention Barreto et al. (2012); the interface is numerical, it was developed in Excel Visual Basic, and the trajectories are static images; that is, a numerical development does not generate a schematization dynamically, only the design variables and the elements that comprise it are identified. In Musa et al. (2018), VB.NET was used, considering that directional calculations are tedious, susceptible to errors, and take much time; when they are done manually or in Excel, the trajectories are displayed in 2D and 3D. Likewise, Hernández (2018) developed an interface with Matlab; it displays the "S, J, double angle" trajectories in 2D. Finally, Ichenwo and Olushola (2018) used MS-Excel for their interface, in which they display the "increase and maintain" trajectory in the cases $R > d$ and $R < d$.

In the case of GeoGebra, the initial points of the directional trajectory, vertical section, center, target/objective section, radius, and construction angle were schematized to later identify the interior angles of each of the triangles that are formed, validating their numerical values, unlike previous studies, our work facilitates the understanding and application of trigonometry in directional drilling. The relevance of GeoGebra is that it integrates the points related to the movement, allowing the user to visualize the change in values and angle immediately, maintaining the general visualization of the trajectory, and reducing the design time of the trajectories.

For the above, Girling (2008) considered one of the most interesting challenges in directional drilling to be reducing operating and technological costs, while Millheim (1982) highlights that the oil well design engineer and the field operator maintain direct communication for constant meter-by-meter updates of the drilling development.

Conclusions

In this work, it was possible to integrate the basic knowledge of trigonometry, analytical geometry, and others. The theory of directional trajectories and the GeoGebra program are used to develop an application or tool that schematizes the directional trajectories and determines their numerical variables according to their design conditions.

GeoGebra facilitates the visualization of directional trajectories, regardless of the numerical value of the design variables such as vertical section, construction grade and radius, target section, true vertical depth, and real depth, among others. In the future, this example could be adapted or improved for other particular cases of trajectories, such as “S” and “modified S,” with 3D visualization.

GeoGebra is free software; it can be downloaded and shared, it is multiplatform, and it applies to all educational levels, its limitation corresponds to its two disadvantages: it requires having previous knowledge of mathematics, and it could be confusing for some beginners, its technical needs in computer equipment and cell phones are minimal, unlike other software such as COMPASS (Plan module, used to design oil well profiles or trajectories), DRILLSOFT and these have a cost in the market and are generally only available to oil companies.

Future lines of research

To develop a study of student perceptions of the application of GeoGebra in the design of trajectories in directional drilling in at least four higher education institutions, public and private, in the face-to-face and online modalities, given that contextual conditions continue to change. In this process, qualitative or mixed research approaches could be adopted, both in students and in teachers, since both are the main actors in the teaching-learning process, as part of the development of educational tools that ensure the development of their professional skills.

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