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Scientific articles

Aplicaciones de la derivada mediante un aprendizaje basado en proyectos: un estudio en el bachillerato

Applications of Derivative through Project-Based Learning: A study in the high school

Aplicações da derivada por meio da aprendizagem baseada em projetos: um estudo no ensino médio

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Resumen

Los cursos de cálculo diferencial han sido motivo de constante análisis. De hecho, conocer el origen del conjunto de dificultades que rodean el proceso de aprendizaje ha originado la creación de nuevas estrategias donde exista una metodología activa. Por tanto, el presente estudio incorpora la implementación del aprendizaje basado en proyectos (ABP) como metodología para el abordaje la derivada en problemáticas aplicadas. Para ello, se analizaron en el curso los contenidos a través de prácticas donde los estudiantes modelaron y diseñaron prototipos para resolver una problemática. Para medir el impacto del diseño instruccional se aplicó una metodología cuantitativa donde se comparó el desempeño de los estudiantes de un grupo control, el cual tuvo una metodología pasiva con prioridad en la carga operativa, mientras que el experimental mantuvo el enfoque activo a través del ABP. Los resultados obtenidos mostraron un mejor desempeño en la transición de lenguajes, así como en los procesos de resolución de cada una de las problemáticas por parte del grupo experimental, el cual obtuvo mayores valores en cada indicador analizado. Los aspectos obtenidos abren el camino para la aplicación de nuevas situaciones donde se pueda emplear este tipo de metodologías en otros cursos de matemáticas.

Palabras clave: aprendizaje basado en proyectos, cálculo diferencial, metodología activa.

Abstract

Differential calculus courses have been subject of constant analysis; knowing the origin of the set of difficulties surrounding the learning process has motivated the creation of new strategies where there is an active methodology. The present study incorporates the implementation of Project Based Learning (PBL) as a methodology to approach the applications of the derivative in applied problems. The course contents were analyzed through practices where students modeled and designed prototypes to solve a problem. To measure the impact of the instructional design, a quantitative methodology was applied to compare the performance of the students of a control group, which had a passive methodology giving priority to the operative load, while the experimental group maintained the active approach through the ABPr. The results obtained showed a better performance in the transition of languages, as well as in the process of solving each of the problems of the experimental group, obtaining higher values in each indicator analyzed. The aspects obtained open the way for the application of new situations where this type of methodologies can be used in other mathematics courses.

Keywords: Proyect – Based- Learning, Differential Calculus, Active Metodology.

Resumo

Os cursos de cálculo diferencial têm sido objeto de constantes análises. De facto, conhecer a origem do conjunto de dificuldades que rodeiam o processo de aprendizagem tem levado à criação de novas estratégias onde existe uma metodologia ativa. Portanto, o presente estudo incorpora a implementação da aprendizagem baseada em projetos (PBL) como metodologia para abordar a derivada em problemas aplicados. Para isso, os conteúdos do curso foram analisados por meio de práticas onde os alunos modelaram e desenharam protótipos para solucionar um problema. Para mensurar o impacto do design instrucional, foi aplicada uma metodologia quantitativa onde foi comparado o desempenho dos alunos de um grupo controle, que possuía uma metodologia passiva com prioridade na carga operacional, enquanto o grupo experimental manteve a abordagem ativa por meio do PBL. . Os resultados obtidos mostraram melhor desempenho na transição linguística, bem como nos processos de resolução de cada um dos problemas por parte do grupo experimental, que obteve valores superiores em cada indicador analisado. Os aspectos obtidos abrem caminho para a aplicação de novas situações onde este tipo de metodologias possam ser utilizadas noutros cursos de matemática.

Palavras-chave: aprendizagem baseada em projetos, cálculo diferencial, metodologia ativa.

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Introduction

When reviewing the contents of the study plans and programs in Mexico, any foreigner might think that students in that country have excellent academic performance in Spanish and mathematics, given that these subjects are taught continuously from preschool to middle school. superior. However, according to standardized tests such as the Program for International Students Assessment (PISA), Mexicans show levels below the average of the members of the Organization for Economic Cooperation and Development, with basic levels of knowledge or, in some cases, insufficient (PISA, 2018).

In fact, when analyzing the report presented by the National Institute of Statistics and Geography (INEGI), it is observed that the upper secondary level is where the highest school dropout rate is recorded (INEGI, 2020), despite the fact that the high school in Mexico it should contribute to the development of the country by training citizens for social, economic and democratic progress (General Directorate of Baccalaureate of Veracruz [DGB], 2020). The above shows that this educational level is crucial as a background for the training of future professionals in the country, which is why it is essential to strengthen and provide better training to the student body.

In this sense, one of the subjects that has a significant failure rate is Differential Calculus. In this regard, various investigations have been carried out, such as that of Artigue (2003), Barajas *et al* . (2018) and Herrera and Padilla (2020), among others, which detail the circumstances surrounding the problems related to the understanding of the course contents promoted by traditional teaching with a strong operational load, where priority is given to the aspects procedural and algorithmic.

Likewise, when analyzing the failure rate at the higher level for the Differential Calculus subject, high values are observed (Riego, 2013), which becomes a determining factor for young people who study careers related to engineering and exact sciences do not complete their university studies. Elements such as the lack of understanding of concepts and their application in everyday contexts are considered determining aspects of this problem (Bressoud , 2016). Therefore, according to Thompson and Harel (2021), the way in which calculus is taught in the early educational levels is crucial in the academic life of students.

Now, among the incident factors, the strong operational load that usually exists in various differential calculus courses stands out (Herrera and Moreno, 2021). In this scenario, young people do not develop an adequate conception of the mathematical object, which makes it difficult for them to understand the usefulness of what they are studying and its application in everyday life. As a consequence, Prada and Ramírez (2017) determined that calculus students lack the necessary skills to solve contextualized problems, which underlines the importance and need for an adequate transition between everyday and mathematical language.

On the other hand, Moreno and Cuevas (2004) determined that, in situations where the derivative must be used to solve problems related to the use of concavities, students face difficulties in solving such cases. When analyzing the performance of the students during the resolution phases, a high degree of incomprehension is identified derived from the lack of understanding of the origin of the variable (Herrera *et al* ., 2016) and, in turn, the relationship and understanding of the concept of derivative.

For all of the above, this study focuses on two key questions related to the various problems surrounding the understanding and resolution of applied problems: is there a better understanding and resolution of problems using an active learning methodology? Does the project-based learning approach increase the procedural dimensions in solving optimization problems? Considering these aspects, the study aims to analyze the difficulties that students face during the problem-solving process.

Theoretical elements

Teaching processes experienced little variation throughout the 20th century, where a passive methodology based on master classes predominated, with students acting as simple recipients of knowledge. However, with the technological development of recent decades, this trend has been reversed, which began a new period of innovation in educational strategies.

In parallel with the advancement of civilization, significant transformations have taken place in society, from the innovations of the Industrial Revolution to the scope of the Internet and the dynamic interaction that it has generated from its creation. This conception is taken up by researchers through the so-called education 4.0, which implies a digital revolution with a convergence between technology and physical elements (Bañuelos, 2020).

Nowadays, it is common to find various tools and applications that allow users to interact and build new knowledge through this link, which increasingly reduces the barriers between real and virtual environments. Given the advances achieved within the framework of education 4.0, authors such as Sánchez (2019) propose critical routes that suggest a restructuring of educational models, work approaches and contents of each course. Therefore, it is necessary to develop new strategies in which the student is the protagonist of the learning process while the teacher becomes a facilitator.

One of the active methodologies that has experienced significant progress in recent years is project-based learning (PBL), a teaching strategy that encourages the participation of both students and teachers (Barrera *et al* ., 2022). Unlike the passive methodology, which focuses on the teacher, PBL focuses actions on students, as it requires their active participation and role in the training process (Johari and Bradshaw, 2008).

Based on this, classroom activities are developed through long-term projects that start from an initial premise, where the opinions and contributions of the participants are integrated to create a common product (Domènech, 2018). Maldonado (2008), for his part, describes PBL as a comprehensive pedagogical strategy based on collaborative learning so that participants solve a problem through the design and implementation of a project that addresses a specific problem.

According to Tippelt *et al.* (2001), some of the key characteristics of PBL are the following: i) the connection with real situations and its practical relevance, ii) an action-oriented approach and the creation of a product, iii) holistic and comprehensive learning, iv) collaborative participation, and v) interdisciplinary learning. These factors allow us to move the analysis of a topic or content towards a practical approach, where participants can create and build solutions based on models or designs that support their proposals.

The advantage that PBL offers in areas such as mathematics has created scenarios where teachers experience greater participation and collaboration from students . This derives from the creation and innovation of new strategies that teachers have incorporated into their sessions (Macias and Arteaga, 2022). With this new approach, the possibility of developing new courses using PBL in complex subjects with high failure rates opens up to promote better learning and knowledge in students.

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Methodology

The present proposal is a descriptive study that uses a questionnaire to collect data on the skills developed and acquired once the application of the didactic unit has been completed. In this research, we worked with two groups (see figure 1): the first, called *the control group* , followed a traditional approach in teaching calculus (Herrera *et al* ., 2020; Moreno and Cuevas, 2004; Riego, 2013). This approach was characterized by a heavy operational load, lecture teaching, little student interaction, and limited feedback on tasks and activities.

On the other hand, the experimental group received teaching based on the proposed instructional design, which highlighted greater interaction between students, the use of technologies, student-centered approaches and greater reflection on the proposed activities by both the teacher and the teacher. as well as the students.

Figure 1. Characteristics of the control and experimental group

Control group	Experimental Group
• Based on the problems detected in other studies. • Traditional teaching (Magistral) • Strong operational load • Little interaction • Little feedback on tasks and activities	• Grounded in Instructional Design • Student-centered teaching • Use of technology • Interaction between participants. • Reflection and analysis of each activity.

Source: self made

Convenience sampling was used to select participants, given that participants are minors and require signed consent to be present during the study. After collecting the documents, 42 participants were identified in the control group, of which 32 were men and 10 were women. On the other hand, 40 participants were registered in the experimental group, of which 35 were men and 5 women. In order to preserve the privacy of each young person, they were assigned individual keys, starting in the experimental group with the term A1 through A40, and in the control group with the term B1 through B42. The activities carried out during the implementation of the instructional design are detailed below.

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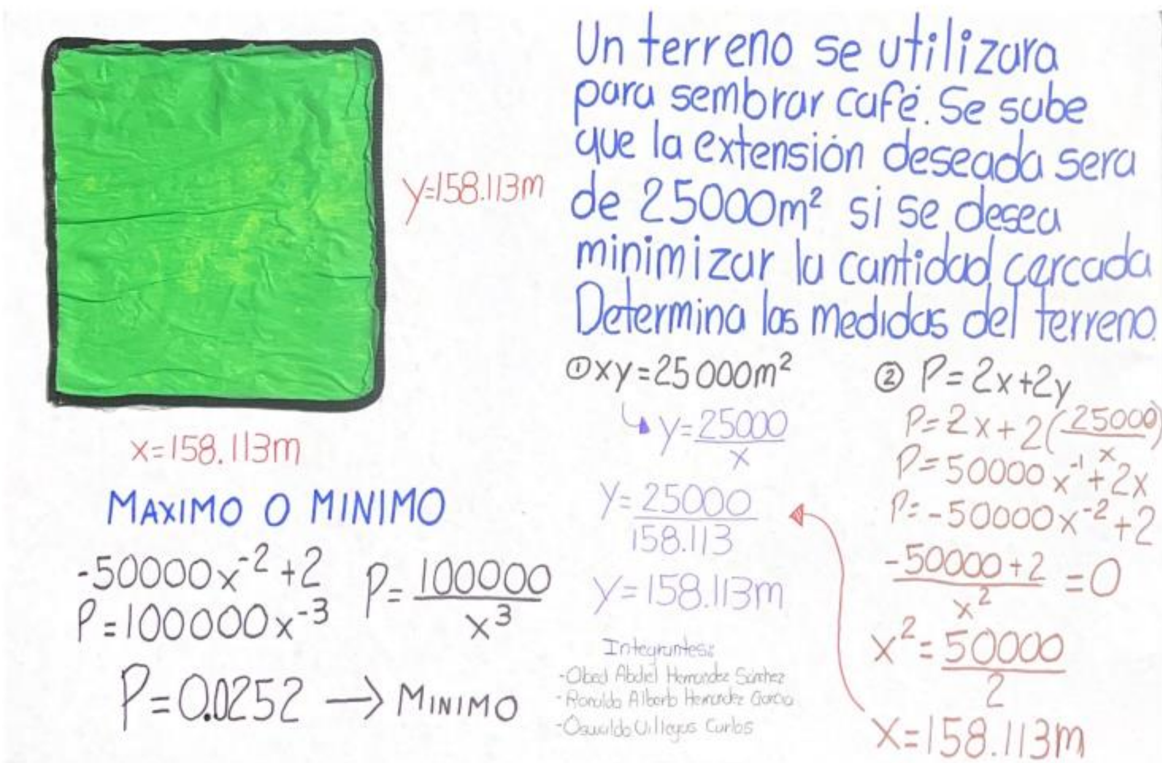
Method

The *derived understanding of the concept* has been the subject of multiple studies (Herrera *et al* ., 2021; Herrera and Padilla, 2020; Moreno and Cuevas, 2004), where the lack of notion of the construct, as well as its application, are detailed. Given these conditions, the present proposal has developed an instructional design where the applications of the derivative will be explored through practices where students will not only observe the procedures on the blackboard, but will also make scale models and graphs that will show their respective findings. .

The first practice consisted of creating a fence with a minimum extension, given the amount of fence available. For this case, the participants used yarn, markers and glue to model the terrain. Each young man, through the use of calculation, determined the measurements his fence should have, as shown in figure 2.

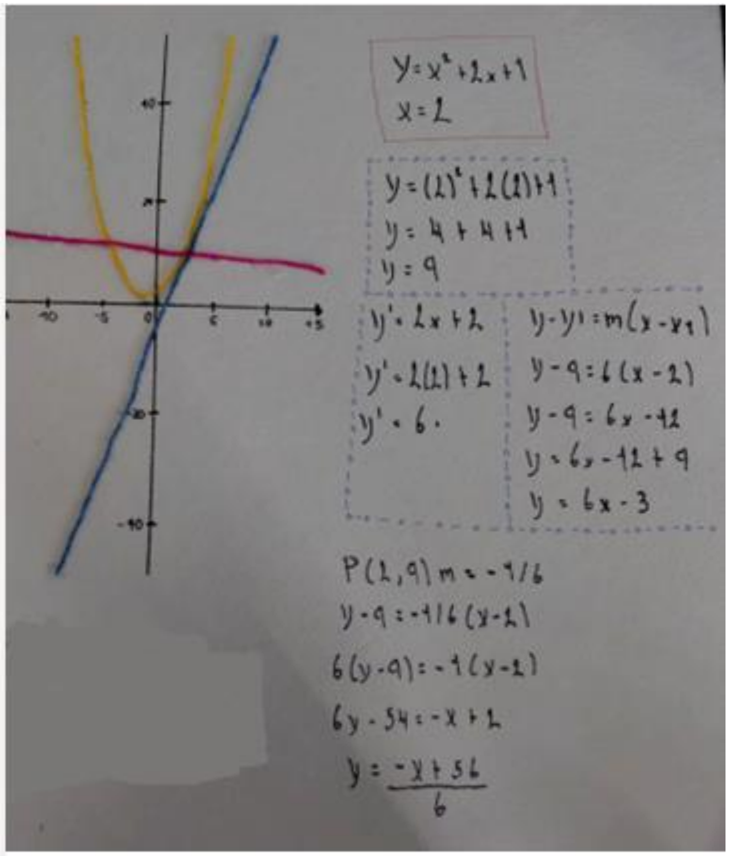
Figure 2. Minimum fencing



Source: Participant A7

For the second practice, the derivative was analyzed as a line tangential to the curve at a given point. In this case, the participants used the same materials to show their progress and argue the solution obtained for the problem posed. It should be noted that in this situation it was not necessary to build a physical model; On the contrary, the young people relied on graphing software to support their results, as seen in Figure 3.

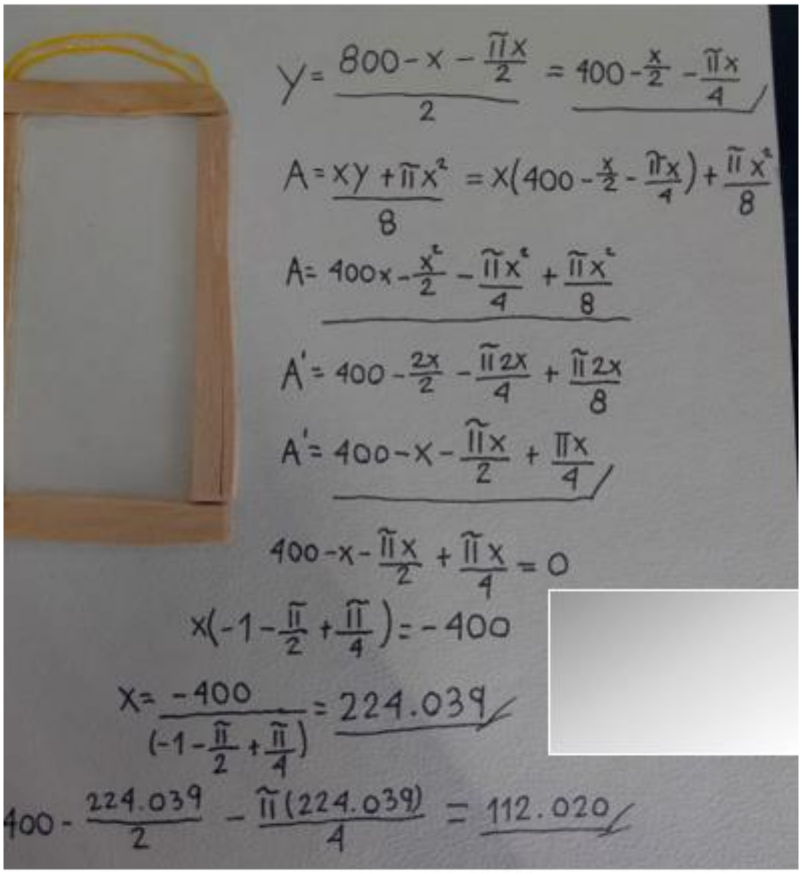
Figure 3. Tangential line



Source: Participant A19

In the third practice, the young people carried out the creation of a Norman window. This problem is common in high school calculus courses, since it appears in textbooks of different subsystems (DGB, 2020), although it should be noted that it is rare to address its analysis through the practical approach to its construction. Given this situation, the participants examined the text and began to develop their proposals to maximize the extension of the structure, as shown in Figure 4.

Figure 4. Norman window



Source: Participant A1

In the last case, the young people were asked to build a box from a square sheet that was provided to each participant. With the material and the problem posed, each participant began to carry out the respective analyzes by using the knowledge acquired during the course to optimize the preparation of the box with the stipulated instructions. On this occasion, the participants presented different designs and proposals for elaboration. In this regard, it is notable that they all agreed on the values and measurements of their model, as shown in Figure 5.

Figure 5. Optimization of a box without a lid

Source: Participant A15

Once the analysis of the proposed practices was completed, any doubts that may have arisen throughout the process were reviewed, as well as any concerns that the participants had about the topics addressed. It is relevant to note that the young people expressed positive opinions during this phase and showed great disposition throughout the various problems raised.

Evaluation instruments

As mentioned, the methodological approach was descriptive, since an open questionnaire was used to analyze the performance of the students of both groups when facing problems in which they applied the knowledge of the differential calculus course through the use of the derivative. The problems posed were extracted from the content of the

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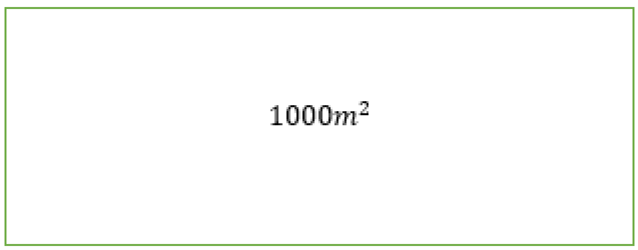
differential calculus book of the General Directorate of Baccalaureate of Veracruz in order to standardize the contents and view both skills from the same perspective.

The questionnaire consisted of four problems that required the use of the derivative to solve the situation posed. The steps taken by the students to solve the optimization problems were analyzed, which involves the execution of certain procedures to obtain the expected response. Below are the problems proposed within the questionnaire:

Problem 1. *In the function* $f(x) = 2x^2 - 5x + 3$*you want to obtain a tangential line when x=-2. Draw a sketch of both graphs.*

Problem 2. *The following piece of land (see figure 6) has an extension* $1000\ m^2$*if you want to fence it, occupying a maximum of its size. Determine the values that meet said criteria*

Figure 6. Problem 2

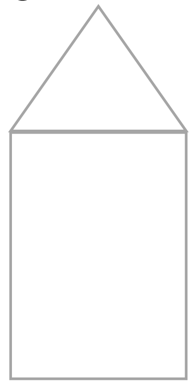


Source: self made

For problem 3, the creation of a window crowned with an equilateral triangle was proposed, for which it was taken into account that the total perimeter was 10 meters. To solve this problem, the relevant functions had to be obtained again, find the inflection points that optimize the desired value and, finally, determine if they correspond to a maximum or minimum to meet the requested requirements.

Problem 3. *A window is going to be built crowned by a right triangle (see figure 7) with a 10m frame. If you want to obtain a maximum area, what measurements should it have?*

Figure 7. Problem 3



Source: self made

Finally, in the last problem it was proposed to optimize the dimensions of a box from a square sheet. This problem is common in several differential calculus courses, where the analysis of a single function is again proposed, in this case, the volume function. Similarly, it is necessary to obtain the critical values and determine which of them corresponds to a maximum by analyzing the first or second derivative. The problem is presented below:

Problem 4. *A box is going to be built from a square sheet of 200cm per side. If the cuts will be made in the corner of value "x", determine the measurement that maximizes its volume.*

Results

To analyze each situation, the breakdown that students must carry out to address the problem is initially presented, starting from a transition from everyday language to mathematical language to reach the solution. These actions consist of i) formulation of the functions of the problem, ii) obtaining the function to be optimized, iii) determining the critical values, iv) identifying the corresponding type of concavity, and v) obtaining the final values that resolve the problem. This information is summarized in a table that indicates the percentage of participants who achieved each item. Finally, the percentages of expected responses for each case are presented.

In the first case, a situation arises in which young people must obtain the tangential line to the curve at a certain point. For this situation, the function is already available, so the first and second solution processes are covered. Therefore, participants had to find the value of the ordinate for the proposed abscissa, then determine the derivative of the function and, thereby, obtain the slope of the line. Finally, with the values obtained, they determine the equation of the point-slope line and sketch the graph that verifies the tangentiality of the line.

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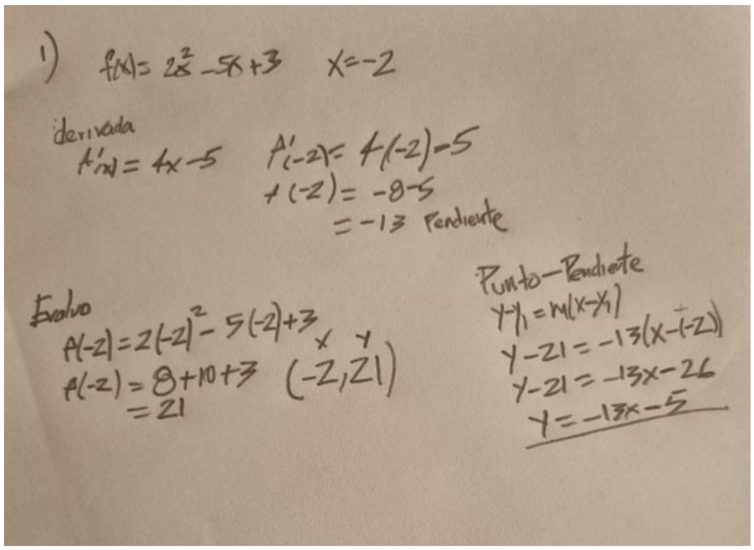
The results obtained show an improvement in the experimental group, where more than 90% of the young people determined the tangential line to the curve at the assigned point (see figure 8). The only area where a decrease was observed in both groups was in the graphic dimension, with 62% success in the experimental group and 19% in the control group. Below are the values obtained:

Figure 8. Resolution of the first problem without graph



Source: Participant A2

Table 1. Tangential line problem

Category	Control group	Experimental group
Obtaining the coordinate of the tangential point	59%	100%
Derivative of the function	95%	100%
Slope of the line	76%	95%
Point-slope equation	47%	92%
Graph sketch	19%	62%
Expected response	Four. Five %	92%

Source: self made

In the second case, a problem arises in which young people must find the dimensions that optimize the fencing of a piece of land. Unlike the first situation, in this case, students must determine the functions of both the perimeter and the area. They then perform inflection

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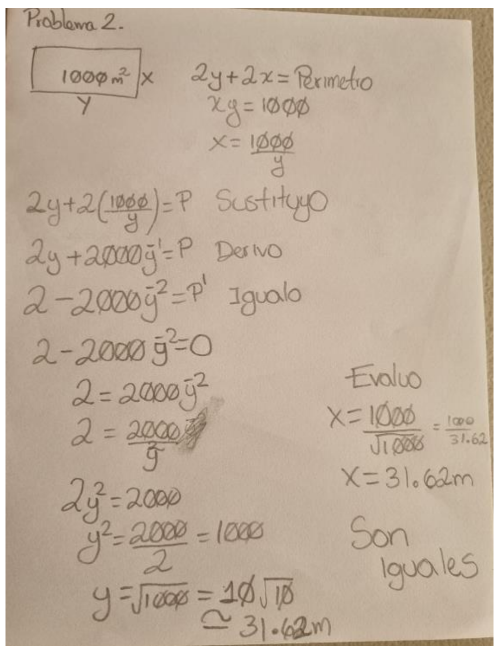
point analysis using the derivative and concavity criteria to determine whether the critical points correspond to a maximum or minimum. Finally, it is important to record the measurements of the terrain and provide a conclusion with the values obtained.

In this situation, a decrease was observed in the different dimensions analyzed. For example, in the control group, 71% correctly proposed the functions of perimeter and area, while in the experimental group it was 95%. In both cases, this process is crucial, since it is the statement of the problem and, if this aspect is not well established, the rest of the solution proposal will not be adequate. Finally, it is observed that only 23% obtained the expected response with all the requested measures in the control group, while in the experimental group the value was 87%, which shows a decrease in both groups with respect to the approach and your response (figure 9).

Figure 9. Resolution problem 2



Source: Participant A4

Table 2. Optimization of a terrain

Category	Control group	Experimental group
Define area and perimeter functions	71%	95%
Derivative of the function to be optimized	61%	95%
Critical points of the function	61%	95%
First or second derivative analysis.	59%	92%
Expected response (Land measurements)	23 %	87%

Source: self made

In the third problem, the participants were presented with a situation similar to that commonly found in DGB books, where the construction of a Norman window crowned by a semicircle is proposed. On this occasion, a crown in the shape of an equilateral triangle was shown. In this case, first the area and perimeter functions must be obtained, which are the basis of the problem statement. Subsequently, the function must be optimized through the derivation process, and then analyze whether the critical points correspond to a maximum or minimum using the first or second derivative methods. Finally, the expected response must be reached with the measurements obtained.

The results showed a notable decrease in both groups, mainly in the approach to area and perimeter functions. This fact affected performance in the other dimensions, since in the control group 23% managed to define the functions and only 7% obtained the expected response. For its part, the experimental group had a decrease from 75% to 62% between the approach and the final expected values. The main difficulty encountered was the lack of skills to determine the area of an equilateral triangle, which on more than one occasion was assigned as the product of the base and the height divided by two, without considering that the equilateral triangle does not have a defined height. , but it must be obtained. The values obtained are presented below.

Table 3. Crowned window

Category	Control group	Experimental group
Define area and perimeter functions	23 %	75%
Derivative of the function to be optimized	14%	75%
Critical points of the function	12%	75%
First or second derivative analysis.	12%	70%
Expected response (Window measurements)	7%	62%

Source: self made

In the fourth problem, participants must pose the volume function, since, since it is a square box with similar cuts, only a single variable will be involved. Subsequently, it must be derived to find the critical values, as well as the type of concavity that corresponds to it. Finally, they must obtain the expected response with the set of measures that allow the optimization of the box.

The values obtained show an improvement in each dimension with respect to the previous problem. On this occasion, in both groups an adequate approach to the function was achieved, which allowed the optimization process to be carried out appropriately. However, despite the notable improvement, a decrease is perceived in both cases when comparing the percentage of problem statement with that of the expected response. In the case of the control group, 90% defined the function and their percentage dropped to 71% in the expected responses, that is, it decreased by 20 points during the process. On the other hand, in the experimental group, the decrease was smaller, since from 100% it was reduced to 92%, having a drop of only 8 points. Each of the indicators are detailed below.

Table 4. Box construction

Category	Control group	Experimental group
Define the volume function	90%	100%
Derivative of the function to be optimized	85%	100%
Critical points of the function	83%	95%
First or second derivative analysis.	83%	95%
Expected response (box measurements)	71%	92%

Source: self made

Discussion

Understanding the concepts of differential calculus is usually a complicated topic for high school and high school students. The difficulties that exist are carried over to the university, where young people are unable to pass calculus courses, which causes, in some cases, school dropout (Riego, 2013). In this regard, it can be stated that the existing high failure rates have been the trigger for the creation of new strategies that improve the performance of young people through technological innovations that allow a new mediation where interaction is a fundamental part of the training process.

Now, when reviewing the results obtained by the control group of this work, characteristics similar to those described by Bressoud (2016) are found, who establishes the various difficulties that surround young people when solving contextualized problems. The values also agree with what was established by Moreno and Cuevas (2004), who focus their research on the analysis of optimization problems, these being the ones used in the present study. In this sense, similarities are obtained in terms of the difficulties experienced by the participants, mainly seen in the control group, where there were cases in which less than half of the young people achieved the expected responses.

Regarding the dimension of the model approach (definition of functions), it was the most recurrent problem found in each of the four situations, an aspect detailed in the study by Herrera *et al* . (2016), who agree that the lack of understanding of the variable does not allow an adequate transition between everyday language and mathematics. This situation was found in both groups, especially in the control group, with cases in which less than half of the participants managed to propose the mathematical model.

Regarding the development of the problems, the analyzed dimensions related to obtaining the derivative, critical points and type of concavity had two aspects. In the first, there is great agreement with what was proposed by Prada and Ramírez (2017), who mention that young people do not have the necessary skills to solve problems, an aspect evidenced in the control group. The second aspect is precisely contrary to what these authors propose, since in the experimental group a substantial improvement was seen in the development of the different dimensions, which reflects both greater skill and understanding of the resolution processes.

Finally, although the course did not have a strong operational load in the experimental group, a great response was achieved in them, since when analyzing the procedural part, a better performance was perceived compared to the control group, the latter being the ones who maintain a strong priority to this type of dimension. This situation agrees with what Herrera *et al proposes* . (2020), who establish that a course properly focused on understanding the object allows for a greater understanding of the concept.

Conclusion

When reviewing the different differential calculus courses that exist in student life, it is observed that the starting point is in high school. The actions carried out at this level impact the future academic development of students, who often arrive at the university with deficiencies in knowledge and skills. Given this situation, it is important to create new strategies and improvement routes that provide more complete training to students, which not only focuses on merely procedural aspects, but also extends to conceptual and attitudinal aspects.

Now, the main objective of this study was the development of a course oriented towards the construction and elaboration of prototypes that would allow students not to remain with the knowledge taught by the teacher. On the contrary, they sought to generate new experiences where they could visualize in real time the way in which their calculations and procedures affected the design of an element or project.

The results obtained indicate significant progress when using this methodology, since the participants developed procedural skills equal to or greater than those of the control group. Addressing the topic under an approach that incorporates more records and strengthens the conceptual notion, along with the procedural one, through practical mechanisms, fostered a more fluid development and with greater certainty in the processes that were carried out throughout. throughout the resolution of the problem.

When analyzing the data in detail, it is observed that there is an improvement in each of the dimensions analyzed. From the formulation of the problem, the development of procedures and obtaining expected answers, which is a factor that is normally not present in differential calculus students at the university and high school level.

Finally, this research contains valuable information about the way in which a differential calculus course can be approached through practical problems and the construction of models or projects. The results allow us to continue studying and reflecting on this strategy in subsequent courses, such as those of integral calculus in high school and those of the university stage. In summary, this proposal represents a first approach towards better development of skills within the students' training process.

Future lines of research

The teaching of the derivative in differential calculus courses has remained under the same strategy for many years. Research shows the great delays that exist due to these actions of teachers. Therefore, it is important to adopt new methodologies that are more focused on students, that involve aspects specific to the context and situation of the environment where the courses are developed. The present proposal allows us to show a new route that provides not only a better understanding of the concepts, but also an alternative for young people to understand the usefulness and applicability of the concepts, leaving them with a better notion of the subject.

At the same time, it is necessary to extrapolate the achievements obtained to the different subjects that make up the disciplinary field of mathematics and even to the differential calculus courses themselves, since it must be considered that each environment is different and the results obtained will allow us to enrich the methodology of the present proposal. Building a more reflective and critical student body is a fundamental pillar for the graduation of young people from high school, in which they will be trained and guided towards their future university and, in some cases, work stage. Finally, this research opens the way for teachers to begin to innovate, build and share their proposals in order to create new active methodologies that are part of the teaching axis of this century.

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