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Artículos científicos

Estrategias didácticas en la enseñanza de los productos notables y la factorización en la telesecundaria

Teaching Strategies in the Teaching of Notable Products and Factoring in the Telesecundaria

Estratégias didáticas no ensino de produtos notáveis e fatoração em telesecundária

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Resumen

El presente trabajo tiene el objetivo de resignificar las operaciones matemáticas de los productos notables y la factorización con procedimientos geométricos y razonamiento lógico matemático, de tal manera que los estudiantes vean las operaciones como inversas una de otra y esto les permita transitar matemáticamente con mayor facilidad. Este contenido matemático se estudia en el tercer grado de educación telesecundaria, en el primer bloque del programa. Para aplicar las actividades didácticas, se rescataron los conocimientos previos de los estudiantes, lo cual les permitió construir con mayor facilidad las reglas para el desarrollo de los productos notables como es el caso de los binomios cuadrados, conjugados y binomios con un término común; posteriormente, se realizaron las actividades inversas, es decir, pasar de un trinomio cuadrado perfecto a un binomio cuadrado y así sucesivamente con los otros dos productos notables. En cuanto a los resultados obtenidos por la implementación de las secuencias didácticas, 17 alumnos (85 %) tuvieron un mejor aprovechamiento escolar. En conclusión, los alumnos pudieron relacionar las operaciones en forma retrospectiva hasta llegar al punto de partida; este razonamiento casi desperdiciado en la enseñanza de la matemática permite apropiarse de la resolución de problemas con su respectiva comprobación, eso quiere decir que existe un aprendizaje con un razonamiento metacognitivo. Las únicas dificultades que se observaron en el aprendizaje se presentaron en la representación gráfica de los binomios conjugados y en la construcción de la regla de los binomios con un término común.

Palabras clave: plan de estudios, resolución de problemas, secuencia didáctica.

Abstract

The present work has the objective of resignifying the mathematical operations of the notable products and the factorization with geometric procedures and mathematical logical reasoning, in such a way that the students see the operations as inverses of each other and this allows them to move mathematically with greater ease. This mathematical content is studied in the third grade of telesecundaria education, in the first block of the program. To apply the didactic activities, the previous knowledge of the students was rescued, which allowed them to more easily build the rules for the development of notable products, such as the case of square binomials, conjugates and binomials with a common term; subsequently, the inverse activities were carried out, that is, going from a perfect square trinomial to a square binomial and so on with the other two notable products. Regarding the results obtained by the implementation of the didactic sequences, 17 students (85 %) had a better school performance. In conclusion, the students were able to relate the operations retrospectively until they reached the starting point; this almost wasted reasoning in the teaching of mathematics allows appropriating the resolution of problems with their respective verification, that means that there is learning with metacognitive reasoning. The only difficulties observed in learning were presented in the graphical representation of the conjugate binomials and in the construction of the binomial rule with a common term.

Keywords: curriculum, problem solving, didactic sequence.

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Resumo

O presente trabalho tem o objetivo de ressignificar as operações matemáticas dos produtos notáveis e a fatoração com procedimentos geométricos e raciocínio lógico matemático, de tal forma que os alunos vejam as operações como inversas umas das outras e isso lhes permita se mover matematicamente com maior facilidade. Esse conteúdo matemático é estudado na terceira série do ensino telesecundário, no primeiro bloco do programa. Para aplicar as atividades didáticas, foi resgatado o conhecimento prévio dos alunos, o que lhes permitiu construir com mais facilidade as regras para o desenvolvimento de produtos notáveis, como é o caso de binômios quadrados, conjugados e binômios com termo comum; Posteriormente, foram realizadas as atividades inversas, ou seja, passar de um trinômio quadrado perfeito para um binômio quadrado e assim sucessivamente com os outros dois produtos notáveis. Em relação aos resultados obtidos com a aplicação das sequências didáticas, 17 alunos (85%) obtiveram melhor aproveitamento acadêmico. Em conclusão, os alunos conseguiram relacionar as operações retrospectivamente até chegarem ao ponto de partida; Esse raciocínio quase desperdiçado no ensino de matemática permite apropriar-se da resolução de problemas com sua respectiva verificação, ou seja, há aprendizagem com raciocínio metacognitivo. As únicas dificuldades observadas na aprendizagem foram apresentadas na representação gráfica dos binômios conjugados e na construção da regra binomial com um termo comum.

Palavras-chave: currículo, resolução de problemas, sequência didática.

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Introduction

It is important that students can navigate through various mathematical registers. As Duval (2016) points out, from a cognitive point of view, there are different semiotic representations and the mathematical object should never be confused with its semiotic representation. Therefore, the objective of this research work is to resignify the mathematical operations of notable products and factorization with geometric procedures and mathematical logical reasoning.

Gómez (2015) starts from the fact that "factorization is a content of the second grade mathematics course, which presents learning difficulties, which affects the development of students in the assimilation of subsequent topics" (p. 25). The same thing happens in the

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In Mexico, the study program designed by the Ministry of Public Education [SEP] (2017) establishes that the topic "Notable products and factorization" (p. 313) be taught in the third grade of secondary education. This topic is not an easy didactic treatment. Normally, students have difficulty assimilating it. Part of its complexity is due to the organizational structure of the programs, lack of updating of the teachers who teach this subject and lack of cooperative work between teachers. According Méndez and Cruz (2008):

> One of the concepts that students do not master in the teaching-learning process, during much of high school, is the development of notable identities and factorization. Many of them do not succeed in solving exercises, nor do they put in place the correct procedures when faced with situations in which this concept is part of the solution. The study of this problem shows a complex didactic phenomenon (p. 59).

Regarding the constructivist current, Rico (1998) mentions that:

- All knowledge is constructed. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction.
- There are cognitive structures that are activated in the construction processes.
- Cognitive structures are in continuous development. Purposeful activity induces the transformation of existing structures.
- Recognizing constructivism as a cognitive position leads to adopting methodological constructivism (pp. 74-75).

The didactic design always seeks that students observe, analyze, build, communicate, argue and solve the problems raised in the aforementioned topics. It is important to mention that teachers can consult texts for preschool, primary, secondary and upper secondary education on the page of the National Commission of Free Textbooks of Mexico (https://www.conaliteg.sep.gob.mx/secundaria .html). Especially, for the teaching of mathematics at the secondary level, it is recommended to consult Ángeles, Guerrero and Loyola (2013) .

The topic treated here corresponds to a particular branch of mathematics.

> Algebra has a great presence as mathematical content at different stages in the educational system, especially from compulsory secondary school to university, although in the last 20 years proposals have emerged to incorporate certain issues of algebraic thinking in primary education. (Socas, 2011, p. 5).

Generally, these subjects are studied in a traditional way in secondary school and high school. In the words of Navarro (2018), "of teachers who transmit and of students who receive information" (p. 195). The teacher offers the factorization formulas so that the students substitute the values in these and obtain the corresponding results. This action, however, does not allow them to build the rules of each binomial, therefore, each time they need to use them, they have to see a form of notable products and factorization.

Here, on the contrary, the aim is for students to be active, not passive, that they are the ones who promote ideas of resolution and that they do not have to wait for the teacher's explanation of the procedure they must follow to solve the problem (del Carmen, Alfonzo and Trejo, 2016). In this line, the teacher must rather fulfill the role of facilitator of learning, not transmitter of knowledge.

Factorization begins in primary education (Alarcón, 2005): when the least common multiple is calculated in the addition and subtraction operations of heterogeneous fractions and also when the greatest common divisor is calculated. In high school, this content is reserved for the third grade (Ángeles, 2013)

The research question of this work is the following: will the students build the rules of the remarkable products with the designed didactic activities? And the hypothesis is that students can build the rules of remarkable products if they analyze each of their parts as multiplications of the algebraic operations.

Methodology

The research is qualitative, exploratory and explanatory. It is about knowing the behavior of the students, clarifying the teaching-learning problems of the notable products and factoring and exploring and explaining the causes of the lack of comprehensive mastery of the contents so that, subsequently, the pertinent measures are taken.

For the elaboration of the didactic sequence, the following procedure was followed:

a) Textbooks for telesecundaria, general secondary, technical and high school (telebachillerato) were reviewed.

b) Through Google and Google Scholar, books in digital format were consulted that could provide more information on the subject in question.

c) Class notes and plans to deal with said content.

d) A possible index was designed to monitor the activities.

e) Some graphic software was selected to support the design of the didactic sequence.

f) The didactic sequence was designed and applied.

g) The results of the application were evaluated.

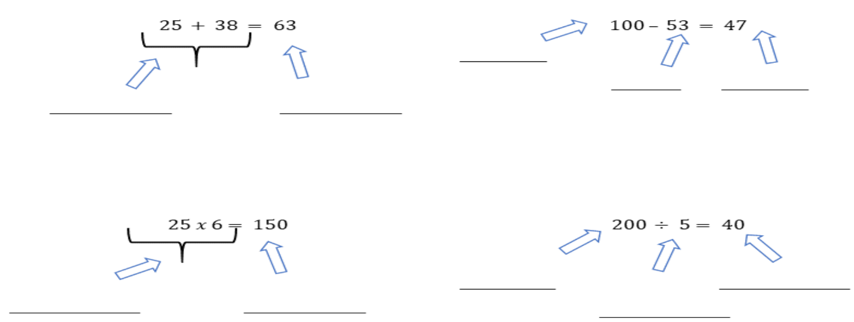
h) It is reported in this document.

Remarkable products

Prior knowledge

- Activity 1. Write the elements of the following operations

Figura 1. Elementos de las operaciones aritméticas



Fuente: Elaboración propia

Laws of exponents

Activity 2. With the multiplication formula $x^m\, x^n = x^{m+n}$, make the following algebraic expressions:

$$x^{12} \cdot x^3, x^9 \cdot x^{-15}, x^7 \cdot x^8, x^2 \cdot x^{-3}, x^{-5} \cdot x^{-4}, x^2 \cdot x^3, x^{-6} \cdot x^{-9}, x^8 \cdot x^{-3}, x^8 \cdot x^{-3}\ y\ x^{20} \cdot x^{-9}$$

Activity 3. With the formula $\frac{x^m}{x^n} = x^{m-n}$, make the following divisions:

$$\frac{x^{12}}{x^5}, \frac{x^{10}}{x^6}, \frac{x^{-2}}{x^{-4}}, \frac{x^{13}}{x^{-8}}, \frac{x^{16}}{x^{-6}}, \frac{x^{-12}}{x^{-20}}, \frac{x^{28}}{x^{50}}, \frac{x^{19}}{x^5}, \frac{x^{23}}{x^7}\ y\ \frac{x^{-7}}{x^4}$$

Activity 4. Through the formula $x^{-n} = \frac{1}{x^n}$, Change the negative powers to positive powers of the following expressions:

$$x^{-2}, x^{-5}, x^{-9}, x^{-1}, x^{-6}, x^{-15}, x^{-12}, x^{-8}, x^{-20}\ y\ x^{-17}$$

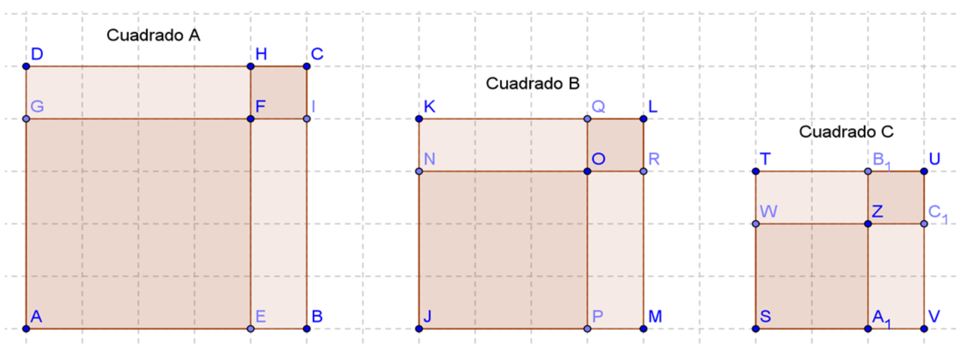
Activity 5. With the formula $(x^n)^m = x^{mn}$, perform the following power operations:

$$(x^2)^3, (x^3)^5, (x^{-2})^{-6}, (x^{-7})^4, (x^5)^3, (x^{12})^{-5}, (x^{-6})^{-4}, \left(x^{\frac{2}{5}}\right)^6, \left(x^{\frac{3}{4}}\right)^{-7}\ y\ \left(x^{\frac{5}{6}}\right)^2$$

Square binomial

Activity 6. Find the area of the following squares by colored parts

Figura 2. Cuadrados de diferentes tamaños

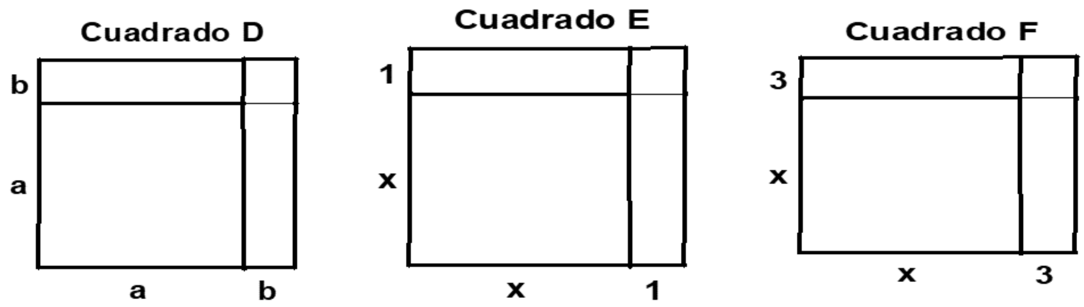


Fuente: Elaboración propia

Area of square A: _____ Area of square B: _____ Area of square C: ______

Activity 7. Calculate the area that has its sides in algebraic terms.

Figura 3. Cuadrados con medidas algebraicas



Fuente: Elaboración propia

Área: ___________________ Área: ___________________ Área: ___________________

Activity 8. Reviewing patterns of regularity, we have the following products, step by step:

- Square D. $(a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$
- Square E. $(x+1)(x+1) = x^2 + x + x + 1^2 = x^2 + 2x + 1$

To analyze, we are going to simplify the binomials as follows:

$(a+b)(a+b) = (a+b)^2$, $(x+1)(x+1) = (x+1)^2$ y $(x+3)(x+3) = (x+3)^2$

We now have the following with respect to square D: the product of the sides is represented as $(a+b)^2$, whose product is $a^2 + 2ab + b^2$

How do you get the following algebraic terms?

- The first term:
- The second term:
- The third term:

Now we have the following with respect to the square IN: the product of the sides is represented as $(x+1)^2$, whose product is $x^2 + 2x + 1$

How do you get the following algebraic terms?

- The first term:
- The second term:
- The third term:

We now have the following with respect to square F: the product of the sides is represented as $(x+3)^2$**,** whose product is x^2+6x+9

How do you get the following algebraic terms?

- The first term:
- The second term:
- The third term:

Write a rule to obtain the product of a binomial squared: "It is the square of the__________________________________".

Activity 9. With the obtained rule, solve the following square binomials:

$(x+5)^2, (x-2)^2,\ (x+9)^2, (2x+4)^2, (3x-2)^2, (3x+7)^2, (5x+10)^2, (5x+10)^2$

Conjugate binomials

Activity 10. Answer the following questions:

- What are symmetric numbers?
- What are conjugate binomials?

Activity 11. Determine the shaded area of the strongest color of the following geometric figures.

Figura 4. Cuadrados para binomios conjugados

Figura G

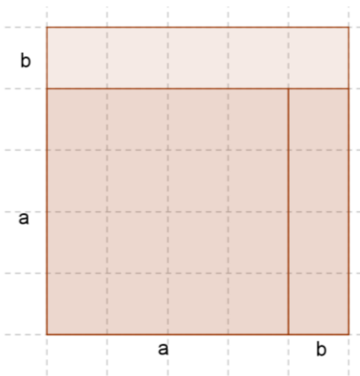
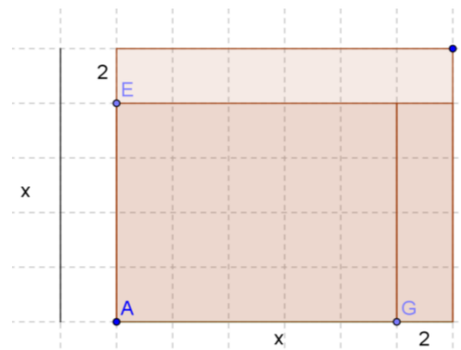


Figura H



Fuente: Elaboración propia

- What algebraic expression corresponds to the base of figure G?
- What algebraic expression corresponds to the height of figure G?
- What is the algebraic expression in terms of area of figure G?
- What algebraic expression corresponds to the base of figure H?
- What algebraic expression corresponds to the height of figure H?
- What is the algebraic expression in terms of area of figure H?

Activity 12. Reviewing patterns of regularity, we have the following products, step by step:

- Square D. $(a+b)(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$
- Square E. $(x+2)(x-2) = x^2 - 2x + 2x - 4 = x^2 - 4$

From figure G we have the following observations: its sides measure $(a+b)(a-b)$, have a common term (a), have two symmetric terms ($b\ \ y\ -b$), its product is made up of a difference of two quadratic terms, one of them being the common term (a^2) and the other is the symmetric term (b^2).

How do you get the first two algebraic terms?

From figure H we have the following observations: its sides measure $(x+2)(x-2)$, have a common term (x), have two symmetric terms (2 and -2), their product is made up of a difference of two quadratic terms, one of them is the common term (x^2) and the other is the symmetric term (4).

How do you get the first two algebraic terms?

Write a rule to obtain the product of the conjugate binomials: "It is the square of the ____"

Activity 13. Graph the following conjugated binomials in your notebook and find the area of each of them, applying only the rule obtained. These are:

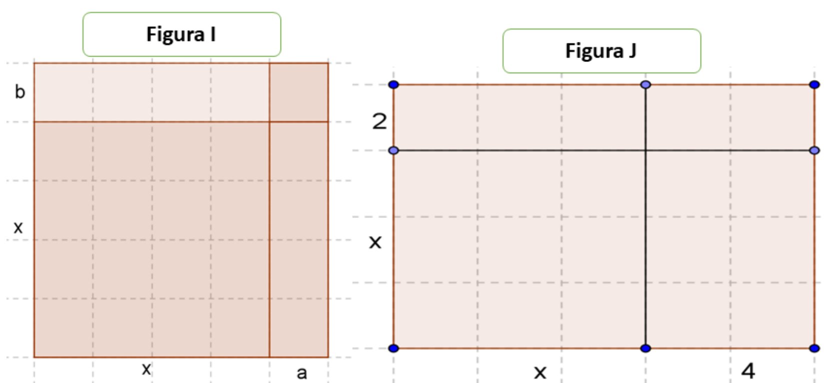
$$(x+1)(x-1), (x+3)(x-3), (x+5)(x-5), (x+8)(x-8), (x+10)(x-10)$$

Binomials with a common term

- What is meant by binomial and by common in mathematical terms?

Activity 14. Determine the sides and the area of each of the parts that make up the following geometric figures.

Figura 5. Cuadrados que conforman binomios con un término común



Fuente: Elaboración propia con base en Steven (2013)

- What algebraic expression corresponds to the base of figure I?
- What algebraic expression corresponds to the height of figure I?
- What is the algebraic expression in terms of area of figure I?
- What algebraic expression corresponds to the base of figure J?

- What algebraic expression corresponds to the height of figure J?
- What is the algebraic expression in terms of area of figure J?

According to what has been observed, we can conclude that the product of two binomials with a common term is obtained as follows:

- The first term:
- The second term:
- The third term:

Write a rule to obtain the product of the binomials with a common term: "It is the square of the ____".

Activity 15. Graph the following binomials with a common term in your notebook and find the area of each of them, applying only the rule obtained. The binomials are the following: $(x+2)(x+1)$, $(x+3)(x+5)$, $(x+5)(x+4)$, $(2x+8)(2x+1)$, $(3x+10)(3x-4)$

Factoring Algebraic Expressions

The perfect square trinomial

Activity 16. Answer the following questions:

- What is meant by factoring?
- From which binomials are the perfect square trinomials generated?

Now we know that a factored perfect square trinomial corresponds to a square binomial. An example:

a) $a^2 + 2ab + b^2 = (a+b)^2$

- How do you get the first term of the square binomial?
- How do you get the second term of the square binomial?
- How do you get the sign that separates the obtained roots?

Summary of the procedure: given the perfect square trinomial, proceed as follows:

$$a^2 + 2ab + b^2$$

Step 1. Take the square root of the quadratic terms.

$$\sqrt{a^2} = a, \sqrt{b^2} = b$$

Step 2. Both roots are separated by the sign of the second term of the trinomial.

Step 3. Enclose the two signed roots of the second term of the perfect square trinomial in parentheses and square it.

Step 4. The expression is as follows:

$$(a + b)^2$$

Activity 17. Factor the following trinomials (identify what they are):

$$4x^2 + 24x + 36, 25x^2 - 20x + 4, 9x^2 + 24x + 16, 49x^2 + 29x + 4$$

The difference of squares

Activity 18. Answer the following questions:

- What do you understand by difference?
- What do you understand by squares?

Look at the following equalities:

$$(a + b)(a - b) = a^2 - b^2$$

- How do you think the common term (a) is obtained?
- How do you think you get the symmetric term (*b y -b*)?

Based on what you mentioned, write a rule to go from a difference of squares to two conjugate binomials: "The square root of ____".

Activity 19. Using the rule described above, factor the differences of squares of the following algebraic expressions:

$$x^2 - z^2, 9x^2 - 16y^2, 49x^2 - 64, 4x^2 - 81, 25x^2 - 36$$

Binomials with a common term

Activity 20. Answer the following questions:

- Do you remember how to obtain a second degree trinomial?

$$(x+3)(x+4) = x^2 + 7x + 12$$

The first term is called the quadratic term (x^2), the second is called the linear term (7x) and the third independent or constant term (12); while the elements of the binomials are: common term (x) and non-common terms (3 y 4)

Answer the following questions

- With the elements of the second degree trinomial, how do you get the common term?
- With the elements of the second degree trinomial, how do you get the linear term?
- With the elements of the second degree trinomial, how do you get the constant?

With the previous reasoning, a rule is established to factor the second degree trinomials:

$$x^2 + 7x + 12$$

Step 1. Two parentheses are opened:

$$x^2 + 7x + 12 = (\quad\quad)\ (\quad\quad)$$

Step 2. The square root of the quadratic term is taken and placed at the beginning of each parenthesis.

$$\sqrt{x^2} = x, \qquad\qquad x^2 + 7x + 12 = (\,x + \quad\quad)\ (\,x+ \quad\quad)$$

Step 3. Two numbers are sought that when added together give the linear term and the same multiplied by each other is equal to the constant.

$$(\quad 3 \quad) + (\quad 4 \quad) = 7$$

$$(\quad 3 \quad)\ (\quad 4 \quad) = 12$$

Step 4. These red numbers are placed in the parentheses, and are as follows:

$$x^2 + 7x + 12 = (x + \ 3 \ ) \ (x + \ 4 \ )$$

With the previous procedure, factor the trinomials of seconds indicated below:

$$4x^2 + 14x + 12, 9x^2 + 27x + 20, 9x^2 + 9x + 2, 49x^2 + 56x - 20, 4x^2 - 8x - 45$$

$$36x^2 + 6x - 6, 25x^2 + 20x + 3, 9x^2 + 36x + 20, 64x^2 + 24x - 10, 100x^2 - 60x + 8$$

Intervention results

They can be considered from good to excellent, since the normalist students understood the two subjects, notable products and factorization, which are studied in the first block of the third grade in telesecundaria. If mathematical competencies in secondary education are taken into account:

a) Solve problems autonomously.

b) Communicate mathematical information.

c) Validate procedures and results.

d) Manage techniques efficiently.

The four mathematical competencies were achieved with normalist students.

Tabla 1. Rúbrica de productos notables y factorización

Criterios de evaluación	Niveles de desempeño			
	Excelente	Bueno	Suficiente	Insuficiente
Resolver problemas de manera autónoma	• Realiza algebraicamente operaciones de multiplicación y división. • Maneja las leyes de los exponentes. • Resuelve problemas de productos notables y factorización. • Construye las fórmulas para su aplicación en donde corresponda.	• Resuelve la mayoría de las operaciones algebraicas. • Maneja las leyes de los exponentes. • Resuelve problemas en su mayoría los productos notables y factorización. • Construye las fórmulas para su aplicación en donde corresponda.	• Tiene dificultades para realizar operaciones de multiplicación y división algebraica. • Maneja algunas de las leyes de los exponentes. • Resuelve algunos problemas de productos notables y factorización. • Tiene dificultades para construir las fórmulas de los productos notables y factorización.	• No puede realizar algebraicamente operaciones de multiplicación y división. • Se confunde en el manejo de las leyes de los exponentes. • Resuelve solo algunos problemas de productos notables y factorización. • No construye las fórmulas para su aplicación en donde corresponda.
Criterios de evaluación	Niveles de desempeño			
	Excelente	Bueno	Suficiente	Insuficiente
Comunicar información matemática	• Menciona la manera de cómo realizan	• Menciona la manera de cómo realiza	• Menciona la manera de cómo realiza	• Menciona escasamente la manera de

	las operaciones de las leyes de los exponentes. • Comunica la forma de resolver los binomios cuadrados, binomios conjugados y binomios con un término común. • Comunica la manera de cómo se factorizan los productos de los binomios antes mencionados.	las operaciones de las leyes de los exponentes. • Comunica la forma de resolver los binomios cuadrados, binomios conjugados y binomios con un término común. • Comunica la manera de cómo factorizar algunos de los binomios antes mencionados.	las operaciones de algunas leyes de los exponentes. • Comunica la forma de resolver de algunos binomios. • Comunica la manera de cómo se factorizan los productos de algunos binomios antes mencionados.	cómo realizan las operaciones de las leyes de los exponentes. • Comunica en forma confusa la forma de resolver los binomios. • Tiene dificultades para comunicar lo que realiza.
Criterios de evaluación	Niveles de desempeño			
	Excelente	Bueno	Suficiente	Insuficiente
Validar información matemática	• Comprueba los resultados de las leyes de los exponentes. • Comprueba los productos notables. • Comprueba la factorización.	• Comprueba los resultados de las leyes de los exponentes. • Comprueba algunos casos de productos notables.	• Comprueba los resultados de algunos casos de las leyes de los exponentes.	• Tiene dificultades para comprobar los resultados de las leyes de los exponentes. • Tiene dificultades para

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	Comprueba los productos notables y la factorización por métodos geométricos.	• Comprueba algunos casos de factorización. • Comprueba los productos notables y la factorización por métodos geométricos de algunos.	• Comprueba algunos productos notables. • Comprueba algunos casos de factorización. • Comprueba algunos casos de los productos notables y la factorización por métodos geométricos.	comprobar los productos notables. • Escasamente comprueba la factorización. • Carece de comprobación de los productos notables y la factorización por métodos geométricos.
Criterios de evaluación	Niveles de desempeño			
	Excelente	Bueno	Suficiente	Insuficiente
Manejar técnicas eficientemente	• Conoce el material de geometría. • Construye las figuras geométricas. • Mide adecuadamente. • Posee conocimientos aritméticos. • Maneja adecuadamente la calculadora científica.	• Conoce el material de geometría. • Construye la mayoría de las figuras geométricas. • Mide adecuadamente. • Posee conocimientos aritméticos. • Maneja suficientemente	• Conoce el material de geometría. • Construye la mayoría de las figuras geométricas. • Mide algunas veces adecuadamente. • Posee conocimientos aritméticos bajos.	• Conoce el material de geometría. • Construye la mayoría de las figuras geométricas. • Mide adecuadamente algunas veces. • Posee conocimientos aritméticos insuficientes.

		la calculadora científica.	• Maneja solo algunas funciones de la calculadora científicamente.	• Desconoce el manejo de la calculadora científica.

Fuente: Elaboración propia

Discussion

The objective of Osorio's work (2008), namely, "to find meaning in the equivalence relations between the expressions that refer to the measurement of the figures (congruence between lengths, equivalence between magnitudes) and understand those meanings" (p. 3), led him to the design of manipulable materials for his students to model geometric figures. While in this work the procedure was to link the geometric part with the algebraic part, then obtain the product of several pairs of binomials, to build the rule that will govern and thus obtain the product.

Gómez (2015), for his part, affirms that the student learns to factor when parallel text and problem solving are implemented, since "he expresses in writing what he understands in his reading, solves and solves factorization problems" (p. 5).

Regarding notable products, Barreto (2009) deals with geometric figures such as squares and rectangles to represent the binomials that will be multiplied to calculate the area.

There are works whose approach is analogous to the one applied here but put into practice in other contexts. For example, for Gratian and Aké (2017):

> The interest is the search for contributions for the training of teachers and the improvement of practice in the classroom. In this regard, the results obtained through an open response questionnaire rescue inconsistencies mainly in the specialized knowledge of the content of notable products, but areas of opportunity for their strengthening are also recognized. (p. 1320).

Likewise, the design of the didactic sequences of Tobón (2017) are assertive for the achievement of the objective of the present research work, since the three phases were taken into account: in the first phase, arithmetic or algebraic contents were handled that were used in a second phase, where the mathematical contents were addressed. Thus, it is about giving the cognitive bases to carry out the multiplications, which were carried out in the first three topics.

In addition, collaborative learning, according to Maldonado (2007), is "a social process that is built in interaction not only with the teacher, but also with classmates, with the context and with the meaning assigned to what is learned". learn" (p. 265).

For this reason, it was promoted throughout the didactic sequence, which was favorable to learning, because some students stood out more than others, as usually happens in most classes, and this was used to act as group monitors and support their peers, since they speak the same "technical" language.

It is considered important to mention that the limitations of the study refer to the application of contextual problems, since the didactic treatment was given within formal mathematics, which allows laying the foundations for problem solving; Regarding the areas of weaknesses, it is possible to mention the time allocated in the curriculum for the teaching of mathematics, since there are only two courses of the total subjects of the degree.

Conclusions

Where more cognitive doubts arose was in the geometric representation of the conjugated binomials, since negative terms were handled there. Of the 20 students with whom we worked, only three of them had some cognitive difficulties, this is due to their weak arithmetic background, so during their professional career they must strengthen these weak points to carry out their professional practices and be excellent professionals. .

With this application of sequenced activities, students in training realized that the knowledge studied in other educational levels can really be analyzed back and forth. In this case, when they developed a square binomial, what they obtained was the perfect square trinomial, from which, when factored (expressing a sum in a product), the square binomial is obtained; the same happens with the other binomials: when obtaining their product and then factoring it, we return to the first algebraic expression.

They not only understood the remarkable products from the algebraic point of view, but also geometrically and in their verbal form; this means that the mathematical competencies, as mentioned in the syllabus, were achieved.

On the other hand, it was interesting that they answered at the beginning of each binomial what they knew about the subject and then contrasted this with what they learned at the end of the sequence. One of the contents that was easier for them to obtain the rule was that of conjugate binomials, since they only multiplied the common and symmetrical terms to arrive at the result, which also has its specific name, difference of squares.

The binomials with which they had the greatest difficulty were those that have a common term, these produce a second degree trinomial with different characteristics from the previous ones.

From a personal point of view, it is important to continue doing this type of work for the benefit of the students. The function that was carried out by the person who designed this sequence was that of facilitator of learning: the normalist students were constructors of their knowledge, in such a way that at the end of each case, they concluded with the rule that helps to solve the products or the problem. factoring.

The important thing about this sequence is the reasoning that is generated when working with students in groups, in teams and individually; a favorable learning environment is generated so that students ask questions with due confidence, because it is known that they will be the future teachers.

Finally, future lines of research can be given in the approach of problems that involve the use of notable products and factorization, in order to solve professional problems from the point of view of economics.

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