

Conectando los Espacios de Trabajo Aritmético y Geométrico a través de la noción de aproximación en Geogebra

Connecting arithmetic and geometric workspaces through the notion of approximation in Geogebra

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Resumen

A través de la presente investigación se analizó la aproximación de medidas en objetos geométricos con Geogebra en el nivel bachillerato, y los resultados mostraron que los estudiantes de bachillerato tienen débiles nociones del concepto de aproximación no obstante sus vastos recursos aritméticos. Dichos resultados permitieron, por un lado, analizar la potencialidad que tienen para el aprendizaje al conectar ideas geométricas y aritméticas y, por otro lado, entender mejor sus fortalezas y dificultades. Enmarcamos este trabajo en la teoría de representaciones de Duval (1993), así como en los trabajos de Núñez y Cortés (2008) sobre Ambientes Tecnológicos Interactivos para el Aprendizaje de las Matemáticas (ATIAM), y de Kuzniak (2012, 2013), sobre la importancia de transitar en diferentes campos de las matemáticas.

Palabras clave: Conectando conceptos de aritmética y geometría, Geogebra, Medición, Geometría.

Abstract

Through this research we analyzed the measures approximation in geometric objects with Geogebra ant high school level, and the results showed that high school students have weak notions of the concept of approximation, however its vast resources of arithmetic. These results permitted, on the one hand, analyze the potential they have for learning by connecting ideas geometric and arithmetic, and, on the other hand, better understand their strengths and difficulties. We frame this work in the theory of Duval representations (1993), as well as in the work of Núñez and Cortés (2008) on Interactive Technological Environments for Math Learning (ATIAM), and Kuzniak (2012, 2013), on the importance of transit in different Areas of mathematics.

Keywords: Connecting concepts of arithmetic and geometry, Geogebra, geometry.

Fecha Recepción: Septiembre 2014

Fecha Aceptación: Noviembre 2014

Introduction

Globalization has done that students now on should have certain competencies that enables them to deal with new situations; to do this, the young requires an integral training based on a constant and dynamic update that adapts to the changes of society. That way, the method of traditional teaching, along with Behaviorism of the 1980s, is no longer what the student requires. In this context, continuous improvement in the use of Information and Communications Technology (ICT) in all areas of our lives, has become priority to innovative methods of teaching and learning. The use of technology has allowed dynamic and interactive management of multiple representative systems, which has impacted on learning that puts more emphasis on the concepts and mathematical understanding. In addition, ICTs have allowed students to better understand the problems and difficulties they face in learning certain mathematical concepts.

First experiences with the use of software in the classrooms of Mathematics (laboratories) was an arduous task, which often brought more disadvantages than benefits, due, among other things, to the high cost of computers. Also developed software was little friendly and transparent, with

long execution times, which produced little useful educational experiences from the point of view of mathematical learning. However, over time both hardware and software is have been updated. The latter has served as a point of support for the development of science and technology. Specialists have adapted it, making it more efficient and specialized for use in specific branches of education; for example, math. The benefits influence on the use of technology in the processes of teaching and learning of mathematics, are considerable. One of them is able to dynamically handle mathematical objects in multiple records of representation within interactive schemas, difficult to achieve with traditional resources not easily manipulated. Thus, the mathematical knowledge obtained through the exploration of objects assumes non-traditional characteristics. Therefore, this research was based on the need to find ways in which high school students, through the use of technological tools and learning activities, generate a conceptual learning about the notions of approximation and estimation in interactive learning environments. Such notions, although they relate to the study of arithmetic, that students study virtually throughout all primary school and part high school in our country, are poorly treated in the curriculum and in the classroom. Students from other countries share this situation, so that various researchers (Chang, evils, Mosier & Ginulates, 2011;) Gooya, Khosroshahi & Teppo, 2011; Hannighofer, Van den Heuvel-Panhuizen, Weirich & Robitzsch, 2011; Smith, Van den Heuvel-Panhuizen & Teppo, 2011) have highlighted the need for activities that promote the development of these skills.

Target

The primary objective of the research was to gain a better understanding of the processes that follow students and mathematical resources they use when working with Geogebra activities approach that naturally seek to combine two basic areas in school mathematics: geometry and arithmetic.

Some additional objectives were:

- Provide well-documented information on the strengths and difficulties of high school students in learning the notion of approximation.
- Generate conceptual learning activities to support the use of technological resources in the mathematics classroom, particularly the use of educational software of dynamic geometry.

To achieve these objectives, the following research questions were posed: What features of Geogebra software will generate a learning environment in which students develop skills to achieve estimate and approximate numerical quantities? What importance does the student to use educational software for learning math? And what strategies students used mathematical tools to deal with the situations in activities?

Theoretical Framework

Much research has been done around the use of technological resources in different fields of school mathematics, such as algebra (Guerrero, Rojano, Mavirikis, & Hoyles, 2011; Filloy, Puig, & Rojano, 2008; Kieran, 2007; Guerrero, & Rivera, 2002; Rojano, 2001), geometry (Kuzniak & Rauscher, 2011), or calculation (Aspinwall, Shaw & Presmeg, 1997), studying the processes and approaching learning with these tools. However, it is also necessary to study what happens in the interactions between the different specific domains of mathematics (Kuzniak, 2012, 2013) in order to understand the overall operation of mathematical work. Kuzniak (2012) points out that this should be a back and forth within the different areas of mathematical work (ETM) that students are presented in the classroom. Such interaction may occur in some areas more naturally; is the case, for example, the geometry and arithmetic, where by the concept of measurement of both concepts can be conjugated ETM.

Particularly in the research reported activity analyzed by high school students in tasks that combine the geometric work with arithmetical work in Geogebra. This combination also suggests an ETM as the backdrop to promote the learning of concepts in another ETM; It is not necessary to "leave" the field of mathematics to try to make sense and motivate learning of them own. Similarly, technological and educational software media that display, manipulate and explore different representations of mathematical concepts (Duval, 1993), have become the ideal tools to promote the integration of different areas of mathematical work (ETM).

In addition to using technology in the classroom is necessary to create real mathematical work environments that promote learning. This research takes ideas from multiple investigations in relation to the generation of collaborative learning environments and individual work. Particularly, there is talk of Technological Environments Interactive Learning of Mathematics (ATIAM), a term coined by Cortés and Nunez (2007) to describe the work in the research on the use of computer technology for teaching and learning mathematics, which also converge current

educational issues and trends such as: the development and use of computer programs, using various Internet platforms and the use of calculators, among others, both from the perspective of teachers (didactic and education) and from the point of view of learning mathematics (cognitive aspects) (Cortés, Guerrero, Morales & Pedroza, 2014).

The use of technology in the mathematics classroom modifies the relationships between the various actors interacting in it; in that sense we have defined a ATIAM as "one (room) generated in space or environment where actors of teaching and learning (teacher and student) and the object of knowledge, interact in an organized manner through a methodology that includes learning activities with the use of technology "(Cortés & Núñez, 2007).

Research conducted around the ATIAM (Cortés & Núñez, 2007; Núñez & Cortés 2008; Núñez 2008) has shown that these technological environments have the potential to foster in students developing skills for building and learning processes mathematical concepts. To create a ATIAM must have: 1) a theoretical proposal of teaching and / or learning; 2) activities which facilitate and encourage the construction of learning; and 3) a methodology of education according to the above (Núñez & Cortés, 2008).

The research reported has to do with learning notions of approximation and estimation. Although, as previously mentioned, using a geometric space work (ETG) and context becomes the focus of the analysis is given in terms of these notions.

Currently the math curriculum basic level focuses on skills development and learning of knowledge through problem solving. This curriculum has been relegated to second place some basics that is necessary to return because they are part of mathematical literacy required by any competent citizen; that is, part of the fundamental knowledge (SEP 2011). One is the ability to make approximations; in everyday life we need to do every day and in many ways: while driving, speed estimates; to go to market, price estimates and measures; We realize that we were wrong when we do an operation or judge the validity of certain quantitative statements. Usually, we make qualitative and quantitative estimates; however, many of the phenomena that affect us have become so complex that we can not perceive directly or try to purely qualitative manner.

While various recommendations are made to generate a functional literacy, numeracy which is fundamental in the current curriculum, the notion of approximation is scarce in situations where students are presented, superimposing it using pocket calculator as an aid in troubleshooting.

This situation is shared by students in other countries; different researchers (Chang, et al 2011;. Gooya, et al, 2011;. Hannighofer, et al, 2011;. Smith, 2011) have highlighted the need to incorporate the curriculum activities that support the development of these skills.

Methodology

For this research were designed and adecuaron three tasks to be used as part of the work: Constructing polygons, Construction of regular polygons using isosceles triangles and the problem of the box. The first of these was applied to a group of 14 high school students and the other two to a second group of students. It should be noted that activities: Building Construction polygons and regular polygons through isosceles triangles covering the same subject, but with different approach and, of course, with different questions, a situation that also gives us the opportunity to make a comparison between the responses of the groups.

The educational intervention research was conducted with high school students Melchor Ocampo, under the Universidad Michoacana de San Nicolás de Hidalgo and Conalep II, Morelia. The first group of 14 students was formed by young people who voluntarily chose to participate in the study. These in turn were divided (voluntarily) into five groups with 2 people, a group of 1 person and a group of 3 people.

In the Conalep II, Morelia, he had different numbers of students participating in each of our activities: activities of polygons 26 students, who in turn were divided into twelve teams of two students and two of a student participated . While for the activity called The problem of the box, involving 13 students were divided into 6 teams of 2 members and 1 January.

This experiment was conducted in the period from May 3 to June 17, 2013. The design and development of it was linked to a conception of teaching and learning supported by collaborative work, characterized by mutual acceptance of shared responsibilities participants in a team or group members in which the roles of teacher and students are fundamental. Also, the design and development of interactive model with a single reflection on which it was required to investigate related.

Throughout the implementation was made available to students at all times:

1. Desktop computer at hand for personal use.
2. Geogebra Software installed on the computer for personal use.

3. A projector where the student was also given an introduction to management Software Geogebra also explain some situations where there was doubt general.

4. Conventional materials (worksheets, worksheets notebooks, whiteboard and nibs).

Because students were unaware of the Geogebra software, were given a brief introduction to management, making major emphasis on the tools used in the development of the activity.

With students Conalep II, Morelia, he worked for three sessions of two hours each, while in high school Melchor Ocampo were given only one two-hour session.

Proposal Narrative

The proposal was to implement three activities in ETG, through the concept of measurement, combined with arithmetic notion of approximation. Then it elaborates one and, for reasons of space, the other comments are made only.

The activity called Construction of regular polygons using isosceles triangles, consists of three sections. The first corresponds to a theoretical foundation where it is expected that the student, through answering a series of questions, to be able to remember some key prior knowledge. The second part of the activity corresponding to tasks using the Geogebra software in which a geometric construction that students have to handle is provided. This is the traditional partition of a regular polygon in congruent isosceles triangles as many as the number of sides thereof. In this students are asked to work with the central angle (the α angle Figure 1) in cases of particularly complex regular polygons, such as 7 sides, wherein said angle can only be approximate because of limitations Computer system; constraints in this case are used to generate a significant activity for students, creating conditions for experimentation processes and allowing exploratory work (Guerrero & Cortés, 2013; Tanguay, Geeraerts, Savoy, Venant, Guerrero & Morales, 2013).

By certain questions included in a worksheet, students handling the angle and measure from side, in a process approach, close to best achieve the polygon is requested.

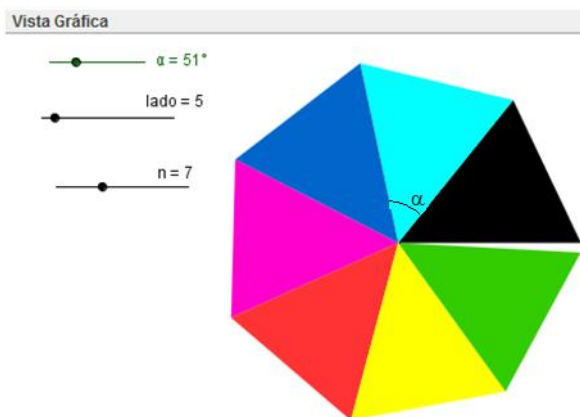


Figure 1. Interactive construction designed for the activity of angular approach

Similarly, it was important that the student obtains the relation of inverse proportionality between the number n of triangles that are needed to build a polygon and the respective angle,

this is $n = \frac{k}{\alpha}$ with constant k and to reflect on the relationship between the angle and length l side.

The third section of this activity gave them the opportunity for students to reach some conclusions on the results obtained, for example, why you can not build a regular polygon with seven sides, or to investigate on other cases of regular polygons and their metric properties, as it is impossible to make a geometric construction by the proposed strategy.

The first of the applied activities, called Building Polygons, also about the possibility or impossibility of constructing regular polygons from triangles. It provides students a file where they can have a set of up to 15 isosceles triangles, all consistent with a previously determined (see Figure 2). With these triangles students are asked to build polygons triangles by joining the vertices and sides. Students can vary the apex angle of the isosceles triangle, as its base and the number of triangles required. In the worksheet for this activity work, questions about how much they should measure the angle at vertex A of triangle ABC to construct regular polygons with different numbers of sides, since the activity is to make the student reflect on the possibility (or not) to construct regular polygons how commonly these figures in textbooks to mention its perimeter or area treated.

Exploración y análisis de polígonos

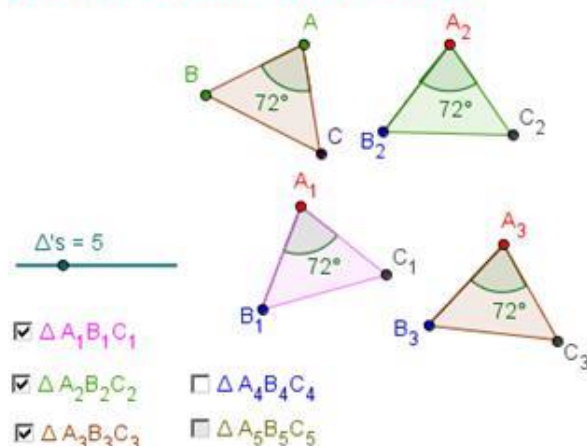


Figure 2. Activity of the file in Geogebra

Another activity called The problem of the box, is a simulation Geogebra in which students have to manipulate certain measures such as the height of the back of a truck passing through a street with a slope (also manipulable) , in order that it can be parked in a shed. It is intended that experimentation (Guerrero & Cortés, 2013) with the virtual model helps students improve their skills approach. It is clear that although the truck has a measure slightly less than the height of the shed, this may not necessarily go without hitting the ceiling. Students must manipulate either the angle of inclination of the road (the angle α Figure 3) or the height of the truck in order to achieve the goal. In a worksheet, are asked to do certain tasks related to the handling of the amounts described to adjust to best measures.

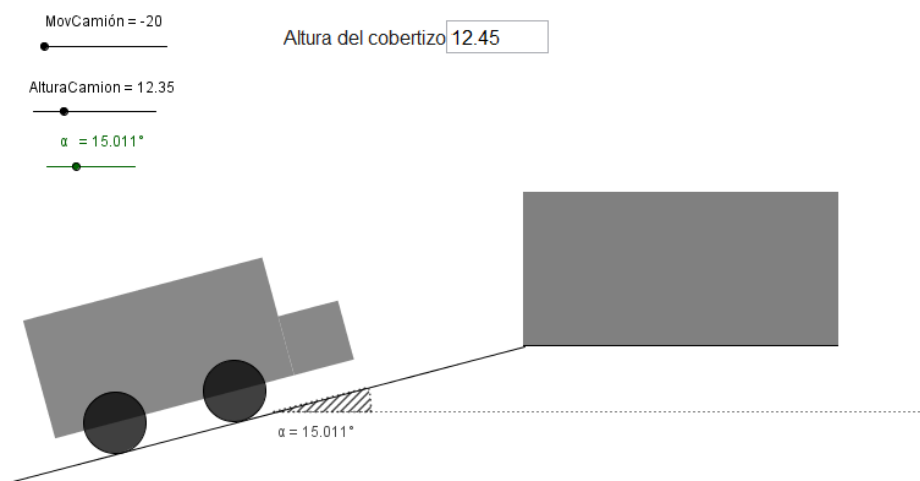


Figure 3. Interactive construction designed to approximate activity

Results

One of the basic characteristics of students who responded activities is the low level of depth of geometrical knowledge, which is considered essential to work in activities and which corresponds to a course of basic geometry. Examples considered in this category are isosceles triangles have two sides equal the sum of the interior angles of any triangle is 180 degrees, the angles in an equilateral triangle are equal, a regular polygon of n sides has exactly n vertices. Similarly, students reported having total ignorance in the use of dynamic geometry software, as Geogebra.

Another important circumstance found by working with students, was that in most situations they respond very simple (naive at times), encompassing a single idea. That is, students were unable to implement and combine several ideas for solving the same problem.

On the other hand, students were able to correctly answer questions about the angles in regular polygons in some cases; for example, in the case of the pentagon measurement indicate that angle α is 72° "are for a total 360° and divided by 5". However, in cases such as the heptagon, students show difficulties because in such cases "do not have an exact value" by dividing the total angle (360°) by the number of sides (7). An answer like this: "The 7-sided polygon is a regular polygon because it lacks closed" shows that students handle concepts only perceptual level.

Table 1 shows some results of student work in relation to the construction presented in the first activity applied, where based on the manipulation of a set of congruent isosceles triangles shown, students are asked to build a regular polygon. A typical response is shown in Figure 4.

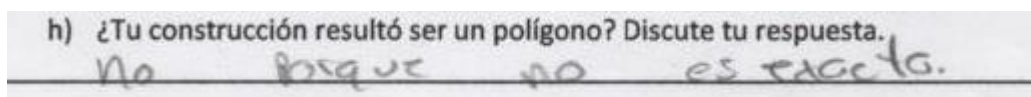


Figure 4. Typical Student Response

As can be seen, students manage to establish themselves the following important result in terms of construction: the impossibility of geometrically construct certain polygons. Could also be observed with confusion regarding some mathematical concepts, eg proportionality.

Table I. Some answers equipment Building activity polygons.

Equipo 1, sec. 424	No, for lack close.
Equipo 2, sec. 424	No, because spare parts in the polygon.
Equipo 3, sec. 424	No, having seven sides although one was smaller.
Equipo 4, sec. 424	No, that is not accurate.
Equipo 1, sec. 403	If, for when we change measures, you change angles and out different figures, regular and irregular polygons.
Equipo 2, sec. 403	No, because it has 7 equal sides and is not proportional.
Equipo 3, sec. 403	Yes, we got the sides.
Equipo 4 y 5, sec. 403	Yup.
Equipo 6 y 7, sec. 403	They did not respond.

For the heptagon, students manage to make approaches to the angle only up to the third decimal place of the α angle without being able to completely close the polygon. They fail to give explanations to this phenomenon and go further approximations. Your answers to questions about the accuracy of results are very basic, encompassing only a perceptual geometric idea without connection to the numerical values obtained.

Conclusions

In mathematics curriculum states that there is a regular polygon of n sides to every integer $n > 2$

and, for each, the central angle is $\frac{360}{n}$ degrees. It is also expected that the student understands that certain fractions can not be expressed exactly in a finite decimal form, and it is

not always possible to use the equal sign between a fraction of the form $\frac{m}{n}$ and a finite decimal notation to represent the same thing. It emphasizes the need to use different forms of writing numbers and that these forms be useful in the subsequent mathematical work; however, these ideas exist in isolation, so they generate conceptual difficulties in student learning as shown by this research.

While students who participated in this research were interested in doing the activities and worked on them throughout all sessions without having difficulties in managing software (even though they said they had never worked with a DMS) showed difficulties to follow a logical argument and ideas were very intuitive and devoid of formalism. Students did not try to check their answers by other means and also contradictory responses were observed in different situations. These difficulties show that students have problems even with very basic notions such as those associated with regular polygons.

Furthermore, the software enabled study Geogebra opening left in some "regular" polygons; making students become aware of this type of construction difficulties. It was an auxiliary important information helped experimentally.

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